AD NUMBER
ADB012040
LIMITATION CHANGES
TO: **Approved for public release; distribution is unlimited.**
FROM: **Distribution authorized to U.S. Gov't. agencies only; Test and Evaluation; OCT 1975. Other requests shall be referred to Air Force Flight Dynamics Laboratory, Attn: FBR, Wright-Patterson, AFB, OH 45433.**
AUTHORITY
AFFDL ltr, 27 Dec 1977

AFFDL-TR-76-3, Vol. II

ADB012040

A NEW FINITE ELEMENT SUPERSONIC KERNEL FUNCTION METHOD IN LIFTING SURFACE THEORY
USER'S MANUAL

LOCKHEED MISSILES & SPACE COMPANY, INC.
HUNTSVILLE RESEARCH & ENGINEERING CENTER
4800 BRADFORD DRIVE, HUNTSVILLE, AL 35807

APRIL 1976

FINAL REPORT 20 OCTOBER 1974 - 20 OCTOBER 1975

Distribution limited to U.S. Government agencies only; test and evaluation statement applied October 1975. Other requests for this document must be referred to AF Flight Dynamics Laboratory (FBR), Wright-Patterson Air Force Base, Ohio 45433.

AIR FORCE FLIGHT DYNAMICS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio 45433

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This technical report has been reviewed and is approved for publication.

Gerald M. Van Keuren

GERALD M. VAN KEUREN, CAPTAIN, USAF
Project Engineer
Optimization Group
Analysis and Optimization Branch

FOR THE COMMANDER

Gerald G. Leigh

GERALD G. LEIGH, Lt. Col, USAF
Chief, Structures Division

Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.

Vol 2 - B011 996

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFFDL-TR-76-3, Vol. II - 2	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A NEW FINITE ELEMENT SUPERSONIC KERNEL FUNCTION METHOD IN LIFTING SURFACE THEORY. Volume 2. User's Manual.		5. TYPE OF REPORT & PERIOD COVERED Final Report. 20 Oct 1974 – 20 Oct 1975
		6. PERFORMING ORG. REPORT NUMBER LMSC-HREC-TR-D496650-II - 2
7. AUTHOR(s) V.Y.C. Young		8. CONTRACT OR GRANT NUMBER(s) F33615-75-C-3001
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lockheed Missiles & Space Company, Inc. Huntsville Research & Engineering Center Huntsville, Alabama 35807		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Project 1370 Task 0401
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Flight Dynamics Laboratory (FBR) Structures Division Wright-Patterson AFB, Ohio 45433		12. REPORT DATE April 1976
		13. NUMBER OF PAGES 63
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 64 p.		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution limited to U.S. Government agencies only; test and evaluation statement applied October 1975. Other requests for this document must be referred to AF Flight Dynamics Laboratory (FBR), Wright-Patterson Air Force Base, Ohio 45433.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

AF-1370   137004

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Finite Element
Kernel Function Method
Lifting Surface Theory
Supersonic Flow

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A new computational method based on the finite element approximation is applied to the kernel function formulation of the supersonic planar lifting surface theory. A computer program developed for the new method is described. The program is applicable to any general planform undergoing harmonic oscillation in a supersonic flow. The input data are the Mach number, reduced frequency and mesh information. The output consists of the lift distributions and a table of the generalized force coefficients.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

210 105

LB

FOREWORD

This report was prepared by personnel in the Engineering Sciences Section of the Lockheed Missiles & Space Company, Inc., Huntsville Research & Engineering Center, Huntsville, Alabama, for the Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio. The research study was performed under Contract F33615-75-C-3001. Capt. Gerald Van Keuren, AFFDL/FBR was the Air Force Project Engineer.

V. Y. C. Young was the principal investigator under the supervision of M. R. Brashears.

The theory for the method used in this computer program is documented as AFFDL-TR-76-3, Vol. I.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

Symbol	
$\bar{C}^{(e)}$	weighted kernel coefficients
f_i	dimensionless modal functions
i	$\sqrt{-1}$
K	kernel function
k	reduced frequency, $\omega s/V$
M	freestream Mach number
$\bar{N}$	shape functions
Q_{ij}	generalized force coefficients
Q'_{ij}	real part of generalized force coefficients in AGARD notation
Q''_{ij}	imaginary part of generalized force coefficients in AGARD notion
x	nondimensional coordinate
x_o	running coordinate in x-direction
y	nondimensional coordinate
y_o	running coordinate in y-direction
$\bar{\lambda}_i^{(e)}$	nodal lift vector at element level

SECTION I

INTRODUCTION

A new finite element supersonic kernel function method in lifting surface theory was presented in Ref. 1. This manual contains the Finite Element Supersonic Kernel Analysis Program (FESKAP), developed for the new method. Descriptions of the main program are presented as well as on the preparation of input necessary to execute the program. A sample run is included to illustrate the usage of the program. Descriptions of each subroutine are presented in Appendix A, and the program listing is contained in Appendix B.

The purpose of the computer program is to generate the generalized force coefficients at one specified Mach number and reduced frequency for a given planform and a given set of modal deflections. The program is applicable to any isolated arbitrary planform in supersonic flow with subsonic/supersonic leading/trailing edges. No thickness effect is accounted for. The unsteady motion is assumed to be harmonic for the analysis.

---

1. Young, V.Y.C., and M.R. Brashears, "A New Finite Element Supersonic Kernel Function Method in Lifting Surface Theory," LMSC-HREC TR D496650, Lockheed Missiles & Space Company, Huntsville, Ala., December 1975.

SECTION II
PROBLEM DESCRIPTION

According to Ref. 1, the finite element formulation of the integral equation in the lifting surface theory is given as

$$\left(\frac{\partial}{\partial x} + ik\right) f_i(x, y) = \frac{1}{4\pi} \sum^{(e)} \bar{C}^{t(e)} \bar{\lambda}_i^{(e)} \tag{1}$$

where k is the reduced frequency, $f_i(x, y)$ is the i^{th} modal function and $\bar{\lambda}_i^{(e)}$ is the column vector containing the nodal lift values of an element, due to a unit displacement in the i^{th} mode. $\sum^{(e)}$ denotes summation over the elements within the forward Mach cone.

The row vector containing the integrated kernel coefficients is defined as

$$\bar{C}^{t(e)} = \iint_{A^{(e)}} \bar{N}^t(x, y) \cdot K(x - x_o, y - y_o)\, dA \tag{2}$$

where $\bar{N}^t(x, y)$ is the row vector of shape functions and K is the kernal function.

The generalized force coefficients are

$$Q_{ij} = -\sum_{i=1}^{N} \iint_{A^{(e)}} f_i(x, y)\, \bar{N}^t(x, y)\, dx\, dy \cdot \bar{\lambda}_j^{(e)} \tag{3}$$

where $\sum_{i=1}^{N}$ denotes summation over all elements.

Equations (1), (2) and (3) form the framework for the computer program development.

SECTION III
PROGRAM DESCRIPTION

The program as presently set up is extremely compact. For example, for a case of 239 nodes constituting 222 elements, the program size is slightly under 20K (decimal) words. Variable dimensions are used in all the subroutines, so that the user needs only to change the first dimension statement in the main program to fit in a new planform. No overlay nor auxiliary file is used in this program.

For convenience, the mode shapes are built into the program. These are represented by the set of 1, x, x^2, y^2, x^2y^2, y, xy or the set of 1, x, x^2, y^2, y, xy. The sets are identified by the number of modes they contain. The user has the option of specifying the first set (NMODE = 7) or the second set (NMODE = 6).

For a given planform and Mach number, the user must first define the characteristic mesh that best fits the planform. The nodes are then numbered starting from left to right with the foremost points and proceeding downstream. The elements are numbered in a similar manner. Fill-in triangular elements with horizontal sides at the supersonic trailing edge are numbered last since they cannot be an influencing element to any collocation point.

Influencing elements within the forward Mach cone of a collocation point are determined automatically by the program. The element containing the collocation point as its most downstream node is defined as the pivotal element. All elements with element number less than the pivotal element number are scanned. Thus all candidates are either forward of or on the same level as the pivotal element. Each in turn is further tested by a logic statement to see if it is within the Mach cone.

As explained in Ref. 1, a table of weighted kernel function coefficients is first tabulated for later table look-up during the solution process. The size of this table is governed by two parameters IMAX and JMAX. IMAX is the maximum number of characteristic elements in the chordwise direction as determined from the mesh. For most planforms, this is given by the number of elements on the centerline. To find JMAX, locate the most extreme collocation point, which is usually the one close to the tip of the trailing edge. JMAX is the number of layers of characteristic elements necessary to cover all the elements within the forward Mach cone.

To take advantage of the symmetry and anti-symmetry, the lift calculation is performed only on the right half of the planform. Each node has its associated mirror image with respect to the centerline. Lift values, with positive or negative sign depending on whether the mode is symmetric or anti-symmetric, are simply substituted in for the mirror node. For consistency, a node on the centerline has itself as the mirror node.

A schematic flow chart of the program is shown in Fig. 1.

<u>Description of Variables</u>

BETA	$\beta = \sqrt{M^2 - 1}$
COEF	Array of integrated kernel function coefficients
IANGLE	Number of angle of element; IANGLE = 3 for triangle and IANGLE = 4 for quadrilateral
IBUF	Buffer array for printing out the list of influencing element number
ICHECK	Option parameter for quick mesh check run
IMAX	Maximum number of regular characteristic elements in the chordwise direction
INFO	Array of element nodal information
JMAX	Maximum number of regular characteristic elements in the Mach line direction
KMAX	Number of element information data cards to be read in

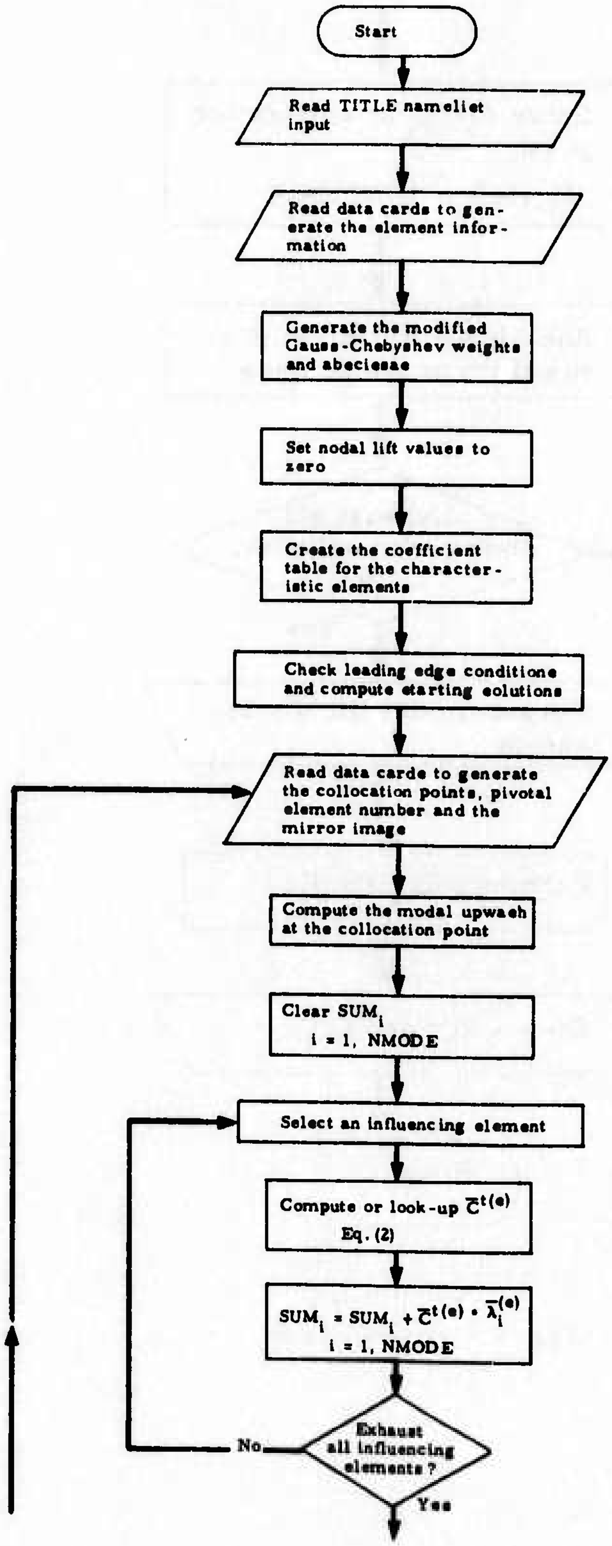

Fig. 1 - Flow Chart for Main Program

(Continued)

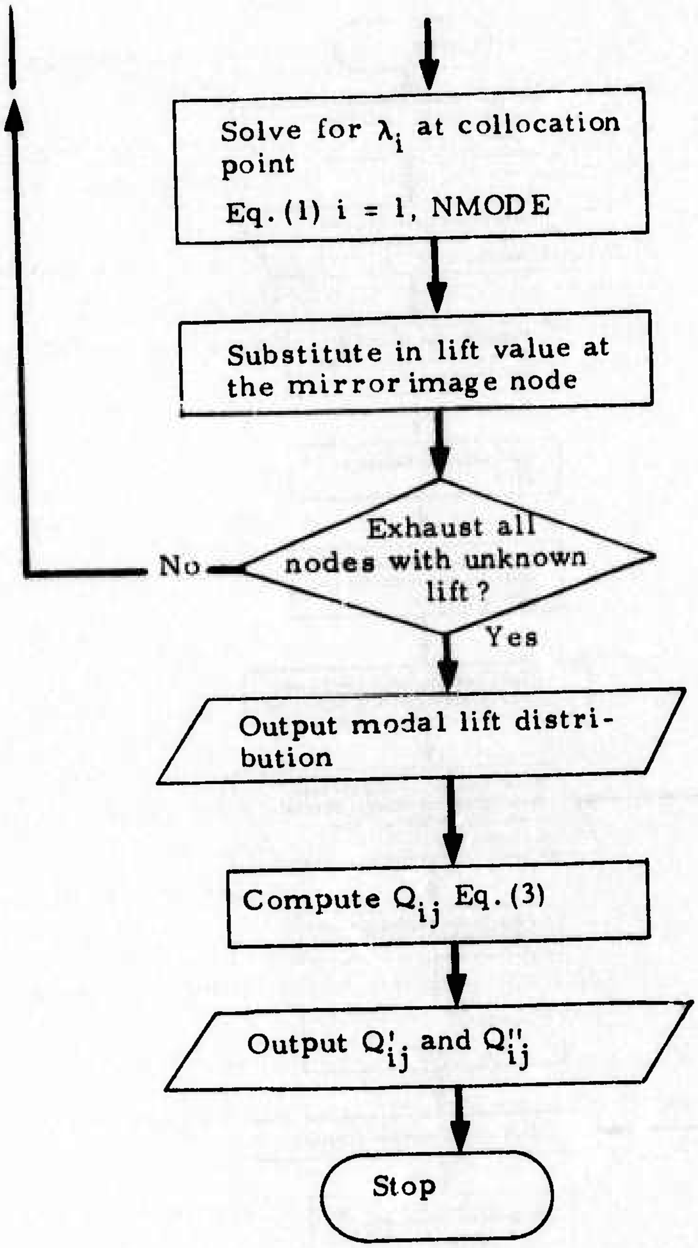

Fig. 1 - (Concluded)

LENODE	Array containing the leading edge nodes
LIST1	Input namelist name
LMAX	Number of entries to the coefficient table; also the number of nodal information data card to be read in
MAXINT	Number of integration points for the modified Gauss-Chebyshev quadrature
MIRROR	Node number of the mirror image point with respect to the centerline
NEL	Pivotal element number
NELEM	Number of elements
NLE	Number of leading edge nodes
NMODE	Number of mode shapes
NP	Number of nodes
QIMAG	Array of Q''_{ij}
QREAL	Array of Q'_{ij}
SWPBK	Sweepback factor
TITLE	Array containing alphanumeric information for identification purposes
TOL	Tolerance, set at 10^{-5} for this program
UPWASH	Array of the upwash
W	Array of the weights of the modified Gauss-Chebyshev quadrature
X	Array of the x-ordinates of the nodes
XEL	Array of the x-ordinates of the elemental nodes
XK	Reduced frequency
XLAMDA	Sweptback angle of the leading edge, in degrees
XLIFT	Array of the lifts at the nodes
XM	Mach number
XO	Relative position in x direction
Y	Array of the y-ordinates of the nodes
YEL	Array of the y-ordinates of the elemental nodes
YO	Relative position in y direction

SECTION IV
INPUT DESCRIPTION

Input cards to this program should be prepared and arranged in the order described below.

A. TITLE CARD (8A10)

Col. 1-80	Information on planform, Mach number, reduced frequency, modes and mesh spacing, for identification purposes

B. NAMELIST Input

$LIST1	
XM	Mach number
XK	Reduced frequency
DELTA	Mesh spacing as measured by the length of the side of the characteristic element
MAXINT	Number of integration points for the modified Gauss-Chebyshev quadrature
DEL	Ratio of the singular strip half width to the element half width
IMAX	Maximum number of characteristic elements in the chordwise direction for the stencil
JMAX	Maximum number of characteristic elements in the Mach line direction for the stencil
XLAMDA	Sweptback angle of leading edge in degrees
NMODE	Number of modes in the set of mode shapes (either 6 or 7)
NP	Number of nodes
NELEM	Number of elements
ICHECK	Option parameter used to check the mesh correctness. For ICHECK = 1, a quick run is performed to print out the element information list, as well as a list of the collocation points with its associated influencing elements. For normal run, this card is to be omitted.

NLE	Number of leading edge nodes
LENODE	Array of the leading edge node numbers
X	Array of x-ordinates of the nodes
Y	Array of y-ordinates of the nodes
$END	

C. Card for Total Number of Element Information Cards to Follow (I5)

D. Element Information Cards (6I5)

These are cards to generate the element number and its nodal numbers in a consecutive manner. Each card begins a new sequence.

Col.	Description
1-5	Number of elements to be generated in this sequence
6-10	Element number of the first element in this sequence
11-15	First nodal number of the first element in this sequence
16-20	Second nodal number of the first element in this sequence
21-25	Third nodal number of the first element in this sequence
26-30	Fourth nodal number of the first element in this sequence (leave blank for triangles).

For example, the card

5 9 22 13 6 12

generates the following information

Element No.	Node Numbers			
9	22	13	6	12
10	23	14	7	13
11	24	15	8	14
12	25	16	9	15
13	26	17	10	16

while the card

1 14 27 18 17

generates the information on a single triangle

Element No.	Node Numbers
14	27 18 17

Element nodes are ordered in a counterclockwise direction, starting with the most downstream node.

E. Card for Total Number of Node Information Cards to Follow (I5)

F. Collocation Point Information Cards (4I5)

These are cards to generate the collocation point, pivotal element number and mirror image node number in a consecutive manner. Each card begins a new sequence.

Col.	Description
1-5	Node number of the first collocation point in this sequence
6-10	Pivotal element number containing the collocation point
10-15	Node number of the mirror image point
16-20	Number of collocation points to be generated

For example, the card

49 30 49 6

generates the following information.

Node	Pivot Element	Mirror
49	30	49
50	31	48
51	32	47
52	33	46
53	34	45
54	35	44

while the card

36 20 35 5

generates the following

Node	Pivot Element	Mirror
36	20	35
37	21	34
38	22	33
39	23	32
40	24	31

SECTION V
SAMPLE RUN

The case of a rectangular planform with $A = 2$, $M = 1.2$ and $k = 0.3$ is used to illustrate the use of this program. The planform with the node numbers and element numbers is set up as in Fig. 2. For a production run, a much finer mesh can be used, and the main program can be dimensioned accordingly. The input deck for this problem is listed on page 17. Some suggested values for the parameters are: MAXINT $\geq$ 12 and DEL = 0.7. These were determined through an accuracy study.

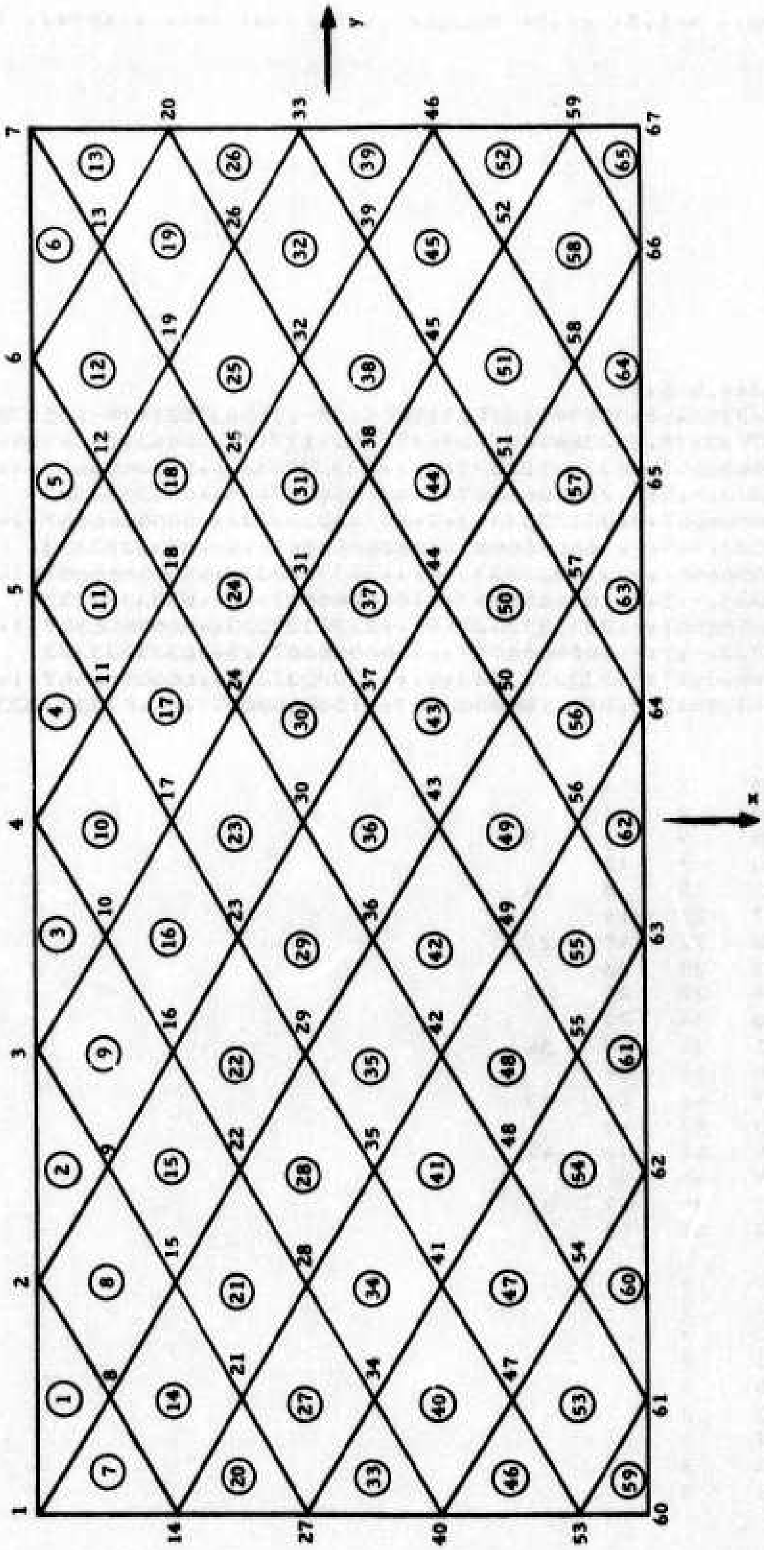

Fig.2 - Mesh for a Rectangular A = 2 Planform at M = 1.2

```
RECTANGULAR A=2., M=1.2, K=.3, MODES= 1, X, X*X, Y*Y, X*X*Y*Y, Y, XY
 $LIST1
 XM = 1.2,
 XK = .3,
 DELTA = .2,
 MAXINT = 15,
 DEL = .7,
 IMAX = 4,
 JMAX = 8,
 XLAMDA = 0.,
 NMODE = 7,
 NP = 67,
 NELEM = 65,
 NLE = 7,
 LENODE = 1,2,3,4,5,6,7,
 X = 7*-.5,6*-.3894458403,7*-.2788916806,6*-.168337521,7*-.0577833612,
     6*.0527707983,7*.1633249581,6*.2738791177,7*.3844332774,8*.4949874371,
 Y = -1.,-.6666666667,-.3333333333,0.,.3333333333,.6666666667,1.,
     -.8333333333,-.5,-.1666666667,.1666666667,.5,.8333333333,
     -1.,-.6666666667,-.3333333333,0.,.3333333333,.6666666667,1.,
     -.8333333333,-.5,-.1666666667,.1666666667,.5,.8333333333,
     -1.,-.6666666667,-.3333333333,0.,.3333333333,.6666666667,1.,
     -.8333333333,-.5,-.1666666667,.1666666667,.5,.8333333333,
     -1.,-.6666666667,-.3333333333,0.,.3333333333,.6666666667,1.,
     -.8333333333,-.5,-.1666666667,.1666666667,.5,.8333333333,
     -1.,-.6666666667,-.3333333333,0.,.3333333333,.6666666667,1.,
     -1.,-.8333333333,-.5,-.1666666667,.1666666667,.5,.8333333333,1.,
 $END
  18
   6    1    8    2    1
   1    7   14    8    1
   5    8   15    9    2    8
   1   13   20    7   13
   6   14   21   15    8   14
   1   20   27   21   14
   5   21   28   22   15   21
   1   26   33   20   26
   6   27   34   28   21   27
   1   33   40   34   27
   5   34   41   35   28   34
   1   39   46   33   39
   6   40   47   41   34   40
   1   46   53   47   40
   5   47   54   48   41   47
   1   52   59   46   52
   6   53   61   54   47   53
   7   59   60   61   53
   9
  11    4   10    3
  17   10   17    3
  24   17   23    3
  30   23   30    3
  37   30   36    3
  43   36   43    3
  50   43   49    3
  56   49   56    3
  64   56   63    3
```

For the initial run, it is desirable to check the correctness of the mesh first. For this purpose, a card containing ICHECK = 1 can be included within the input namelist. This option will bypass all the time consuming computations and output only the mesh information. The user can compare this information against the mesh pattern to eliminate any input error. The output for ICHECK = 1 for this sample run is listed in the following pages.

RECTANGULAR A=2., M=1.2, K=.3, MODES= 1, X, X*X, Y*Y, X*X-Y*Y, Y, XY

MESH CHECK.....

NODE	X-ORDINATE	Y-ORDINATE
1	-.50000+00	-.10000+01
2	-.50000+00	-.66667+00
3	-.50000+00	-.33333+00
4	-.50000+00	.00000
5	-.50000+00	.33333+00
6	-.50000+00	.66667+00
7	-.50000+00	.10000+01
8	-.38945+00	-.83333+00
9	-.38945+00	-.50000+00
10	-.38945+00	-.16667+00
11	-.38945+00	.16667+00
12	-.38945+00	.50000+00
13	-.38945+00	.83333+00
14	-.27889+00	-.10000+01
15	-.27889+00	-.66667+00
16	-.27889+00	-.33333+00
17	-.27889+00	.00000
18	-.27889+00	.33333+00
19	-.27889+00	.66667+00
20	-.27889+00	.10000+01
21	-.16834+00	-.83333+00
22	-.16834+00	-.50000+00
23	-.16834+00	-.16667+00
24	-.16834+00	.16667+00
25	-.16834+00	.50000+00
26	-.16834+00	.83333+00
27	-.57783-01	-.10000+01
28	-.57783-01	-.66667+00
29	-.57783-01	-.33333+00
30	-.57783-01	.00000
31	-.57783-01	.33333+00
32	-.57783-01	.66667+00
33	-.57783-01	.10000+01
34	.52771-01	-.83333+00
35	.52771-01	-.50000+00
36	.52771-01	-.16667+00
37	.52771-01	.16667+00
38	.52771-01	.50000+00
39	.52771-01	.83333+00
40	.16332+00	-.10000+01
41	.16332+00	-.66667+00
42	.16332+00	-.33333+00

43	.16332+00	.00000
44	.16332+00	.33333+00
45	.16332+00	.66667+00
46	.16332+00	.10000+01
47	.27388+00	-.83333+00
48	.27388+00	-.50000+00
49	.27388+00	-.16667+00
50	.27388+00	.16667+00
51	.27388+00	.50000+00
52	.27388+00	.83333+00
53	.38443+00	-.10000+01
54	.38443+00	-.66667+00
55	.38443+00	-.33333+00
56	.38443+00	.00000
57	.38443+00	.33333+00
58	.38443+00	.66667+00
59	.38443+00	.10000+01
60	.49499+00	-.10000+01
61	.49499+00	-.83333+00
62	.49499+00	-.50000+00
63	.49499+00	-.16667+00
64	.49499+00	.16667+00
65	.49499+00	.50000+00
66	.49499+00	.83333+00
67	.49499+00	.10000+01

ELEMENT	NODES			
1	8	2	1	0
2	9	3	2	0
3	10	4	3	0
4	11	5	4	0
5	12	6	5	0
6	13	7	6	0
7	14	8	1	0
8	15	9	2	8
9	16	10	3	9
10	17	11	4	10
11	18	12	5	11
12	19	13	6	12
13	20	7	13	0
14	21	15	8	14
15	22	16	9	15
16	23	17	10	16
17	24	18	11	17
18	25	19	12	18
19	26	20	13	19
20	27	21	14	0
21	28	22	15	21
22	29	23	16	22
23	30	24	17	23
24	31	25	18	24

25	32	26	19	25
26	33	20	26	0
27	34	28	21	27
28	35	29	22	28
29	36	30	23	29
30	37	31	24	30
31	38	32	25	31
32	39	33	26	32
33	40	34	27	0
34	41	35	28	34
35	42	36	29	35
36	43	37	30	36
37	44	38	31	37
38	45	39	32	38
39	46	33	39	0
40	47	41	34	40
41	48	42	35	41
42	49	43	36	42
43	50	44	37	43
44	51	45	38	44
45	52	46	39	45
46	53	47	40	0
47	54	48	41	47
48	55	49	42	48
49	56	50	43	49
50	57	51	44	50
51	58	52	45	51
52	59	46	52	0
53	61	54	47	53
54	62	55	48	54
55	63	56	49	55
56	64	57	50	56
57	65	58	51	57
58	66	59	52	58
59	60	61	53	0
60	61	62	54	0
61	62	63	55	0
62	63	64	56	0
63	64	65	57	0
64	65	66	58	0
65	66	67	59	0

NODE	NEL	MIRROR	INFLUENCING ELEMENTS
11	4	10	4
12	5	9	5
13	6	8	

6

17	10	17	3 4 10
18	11	16	4 5 11
19	12	15	5 6 12
24	17	23	3 4 5 10 11 17
25	18	22	4 5 6 11 12 18
26	19	21	5 6 12 13 19
30	23	30	2 3 4 5 9 10 11 16 17 23
31	24	29	3 4 5 6 10 11 12 17 18 24
32	25	28	4 5 6 11 12 13 18 19 25
37	30	36	2 3 4 5 6 9 10 11 12 16 17 18 23 24 30
38	31	35	3 4 5 6 10 11 12 13 17 18 19 24 25 31
39	32	34	4 5 6 11 12 13 18 19 25 26 32
43	36	43	1 2 3 4 5 6 8 9 10 11 12 15 16 17 18 22 23 24 29 30 36
44	37	42	2 3 4 5 6 9 10 11 12 13 16 17 18 19 23 24 25 30 31 37
45	38	41	3 4 5 6 10 11 12 13 17 18 19 24 25 26 31 32 38
50	43	49	1 2 3 4 5 6 8 9 10 11 12 13 15 16 17 18 19 22 23 24 25 29 30 31 36 37 43
51	44	48	2 3 4 5 6 9 10 11 12 13 16 17 18 19 23 24 25 26 30 31 32 37 38 44
52	45	47	3 4 5 6 10 11 12 13 17 18 19 24 25 26 31 32 38 39 45
56	49	56	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21 22 23 24 25 28 29 30 31 35 36 37 42 43 49
57	50	55	1 2 3 4 5 6 8 9 10 11 12 13 15 16 17 18 19 22 23 24 25 26 29 30 31 32 36 37 38 43 44 50
58	51	54	2 3 4 5 6 9 10 11 12 13 16 17 18 19 23 24 25 26 30 31 32 37 38 39 44 45 51
64	56	63	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21 22 23 24 25 26 28 29 30 31 32 35 36 37 38 42 43 44 49 50 56
65	57	62	1 2 3 4 5 6 8 9 10 11 12 13 15 16 17 18 19 22 23 24 25 26 29 30 31 32 36 37 38 39 43 44 45 50 51 57
66	58	61	2 3 4 5 6 9 10 11 12 13 16 17 18 19 23 24 25 26 30 31 32 37 38 39 44 45 51 52 58

After the mesh has been verified, the program can be run without the ICHECK = 1 option card. The output consists of the arrays of complex lifts due to a unit displacement in the respective mode shapes. The output is formatted such that each line contains four nodal lift values, with the real and imaginary part given in pairs (pages 24 through 27). Following these are the tables of generalized force coefficients (page 28). The upper table represents Q'_{ij} while the lower table represents Q''_{ij}. The output for the sample run is listed in the following pages.

COMPLEX LIFT DISTRIBUTION FOR MODE 1

.00000000	-.90453402+00	.00000000	-.90453402+00	.00000000	-.90453402+00	.00000000	-.90453402+00
.00000000	-.90453402+00	.00000000	-.90453402+00	.00000000	-.90453402+00	-.21542585-01	-.89943505+00
-.21542548-01	-.89943408+00	-.21542567-01	-.89943420+00	-.21542567-01	-.89943420+00	-.21542548-01	-.89943408+00
-.21542585-01	-.89943505+00	.00000000	.00000000	-.41816221-01	-.89677092+00	-.41816209-01	-.89677049+00
-.41816209-01	-.89677049+00	-.41816209-01	-.89677049+00	-.41816221-01	-.89677092+00	.00000000	.00000000
-.29395609-01	-.30085396+00	-.65726056-01	-.89504297+00	-.65726022-01	-.89504258+00	-.65726022-01	-.89504258+00
-.65726056-01	-.89504297+00	-.29395609-01	-.30085396+00	.00000000	.00000000	-.54467405-01	-.46330016+00
-.94982937-01	-.89150189+00	-.94982912-01	-.89150166+00	-.94982937-01	-.89150189+00	-.54467405-01	-.46330016+00
.00000000	.00000000	-.15071793-01	-.11465322+00	-.77853039-01	-.55892447+00	-.12421045+00	-.88626766+00
-.12421045+00	-.88626766+00	-.77853039-01	-.55892447+00	-.15071793-01	-.11465322+00	.00000000	.00000000
-.34142861-01	-.20580967+00	-.10212547+00	-.61970649+00	-.15369800+00	-.87869661+00	-.10212547+00	-.61970649+00
-.34142861-01	-.20580967+00	.00000000	.00000000	-.93298326-02	-.11422743+00	-.54038059-01	-.28172730+00
-.12765897+00	-.65877576+00	-.12765897+00	-.65877576+00	-.54038059-01	-.28172730+00	-.93298326-02	-.11422743+00
.00000000	.00000000	-.19033791-01	-.16937633+00	-.72902214-01	-.34302411+00	-.94309922-01	-.51128176+00
-.72902214-01	-.34302411+00	-.19033791-01	-.16937633+00	.00000000	.00000000	.00000000	.00000000
-.71305386-02	-.10931405+00	-.29715341-01	-.20692647+00	-.30046925-01	-.24927306+00	-.30046925-01	-.24927306+00
-.29715341-01	-.20692647+00	-.71305386-02	-.10931405+00	.00000000	.00000000		

COMPLEX LIFT DISTRIBUTION FOR MODE 2

-.30151134+01	.45226701+00	-.30151134+01	.45226701+00	-.30151134+01	.45226701+00	-.30151134+01	.45226701+00
-.30151134+01	.45226701+00	-.30151134+01	.45226701+00	-.30151134+01	.45226701+00	-.29861880+01	.45576892+00
-.29861847+01	.45576831+00	-.29861851+01	.45576843+00	-.29861851+01	.45576843+00	-.29861847+01	.45576831+00
-.29861880+01	.45576892+00	.00000000	.00000000	-.29642929+01	.46218432+00	-.29642915+01	.46218406+00
-.29642915+01	.46218406+00	-.29642915+01	.46218406+00	-.29642929+01	.46218432+00	.00000000	.00000000
-.98412988+00	.12561527+00	-.29449700+01	.47139149+00	-.29449686+01	.47139117+00	-.29449686+01	.47139117+00
-.29449700+01	.47139149+00	-.98412988+00	.12561527+00	.00000000	.00000000	-.15108425+01	.18084322+00
-.29181809+01	.48047768+00	-.29181801+01	.48047747+00	-.29181809+01	.48047768+00	-.15108425+01	.18084322+00
.00000000	.00000000	-.36790983+00	-.68431429-01	-.18169849+01	.90637580+00	-.28866790+01	.48763924+00
-.28866790+01	.48763924+00	-.18169849+01	.20637580+00	-.36790983+00	-.68431429-01	.00000000	.00000000
-.66016229+00	-.94507711-01	-.20093140+01	.21897143+00	-.28496835+01	.49155262+00	-.20093140+01	.21897143+00
-.66016229+00	-.94507711-01	.00000000	.00000000	-.36742229+00	-.12856196+00	-.90471707+00	-.11319498+00
-.21314424+01	.22337249+00	-.21314424+01	.22337249+00	-.90471707+00	-.11319498+00	-.36742229+00	-.12856196+00
.00000000	.00000000	-.54442368+00	-.22671207+00	-.11041902+01	-.13510789+00	-.16594519+01	-.41861706-01
-.11041902+01	-.13510789+00	-.54442368+00	-.22671207+00	.00000000	.00000000	.00000000	.00000000
-.35035501+00	-.17136233+00	-.66710681+00	-.30966387+00	-.81634627+00	-.42011374+00	-.81634627+00	-.42011374+00
-.66710681+00	-.30966387+00	-.35035501+00	-.17136233+00	.00000000	.00000000		

COMPLEX LIFT DISTRIBUTION FOR MODE 3

.30151134+01	-.22613350+00
.30151134+01	-.22613350+00
.25533169+01	-.24589673+00
.25533201+01	-.24589709+00
.21382550+01	-.28565647+00
.17439839+00	-.11518355+00
.16618637+01	-.33502940+00
.10663632+01	-.39376474+00
.00000000	.00000000
.46001078+00	-.46246660+00
-.13976198+01	-.11302934+00
-.13976198+01	-.11302934+00
-.13701583+01	-.40741469+00
.00000000	.00000000
-.25285223+01	-.24320912+00
-.12987007+01	-.15528215+00
-.27223606+01	-.20759744+00

.30151134+01	-.22613350+00
.30151134+01	-.22613350+00
.25533172+01	-.24589682+00
.00000000	.00000000
.21382551+01	-.28565648+00
.16618637+01	-.33502940+00
.17439839+00	-.11518355+00
.10663626+01	-.39376459+00
-.79696415+00	-.47815948-01
-.37483525+00	-.26866759+00
-.83217483+00	-.33873681+00
.00000000	.00000000
-.13701583+01	-.40741469+00
-.19351452+01	-.14916421+00
-.19351452+01	-.14916421+00
-.27223606+01	-.20759744+00
-.12987007+01	-.15528215+00

.30151134+01	-.22613350+00
.30151134+01	-.22613350+00
.25533172+01	-.24589682+00
.21382565+01	-.28565662+00
.21382565+01	-.28565662+00
.16618624+01	-.33502918+00
.00000000	.00000000
.10663632+01	-.39376474+00
-.37483525+00	-.26866759+00
-.79696415+00	-.47815948-01
-.17146508+00	-.53211994+00
-.10664166+01	-.84534670-01
-.19648511+01	-.17959342+00
-.25285223+01	-.24320912+00
.00000000	.00000000
-.34616042+01	-.10839454+00
.00000000	.00000000

.30151134+01	-.22613350+00
.25533201+01	-.24589709+00
.25533169+01	-.24589673+00
.21382551+01	-.28565648+00
.00000000	.00000000
.16618624+01	-.33502918+00
-.21008564-01	-.19493969+00
-.21008564-01	-.19493969+00
.46001078+00	-.46246660+00
.00000000	.00000000
-.83217483+00	-.33873681+00
-.19648511+01	-.17959342+00
-.10664166+01	-.84534670-01
-.23845225+01	-.28174307+00
.00000000	.00000000
-.34616042+01	-.10839454+00

COMPLEX LIFT DISTRIBUTION FOR MODE 4

.00000000	-.90453402+00
.00000000	-.10050378+00
-.62748775-02	-.23332037+00
-.15849355-01	-.63306928+00
-.20277369-02	-.43623535-01
-.10203176-01	-.65670219-01
-.22948552-01	-.31467749+00
-.29139332-01	-.26179930+00
.00000000	.00000000
-.44317652-01	-.28222599+00
.18365179-01	.85789927-01
.18365179-01	.85789927-01
-.70688208-01	-.29157690+00
.00000000	.00000000
-.37521483-01	-.99938655-01
.12409192-01	-.21013903-01
-.14188962-01	-.57524152-01

.00000000	-.40201511+00
.00000000	-.40201511+00
-.14876610-02	-.33446419-01
.00000000	.00000000
-.66735897-02	-.14326847+00
-.22948552-01	-.31467749+00
-.10203176-01	-.65670219-01
-.18586180-01	-.16273606+00
.17352759-01	.86170861-01
-.16840973-01	-.11341027+00
-.33072399-01	-.17771619+00
.00000000	.00000000
-.70688208-01	-.29157690+00
.21541534-01	.19938403-01
.21541534-01	.19938403-01
-.14188962-01	-.57524152-01
.12409192-01	-.21013903-01

.00000000	-.10050378+00
.00000000	-.90453402+00
-.14876610-02	-.33446419-01
-.20611937-01	-.44219593+00
-.20611937-01	-.44219593+00
-.83438142-02	-.11576757+00
.00000000	.00000000
-.29139332-01	-.26179930+00
-.16840973-01	-.11341027+00
.17352759-01	.86170861-01
-.77130901-01	-.37117156+00
.25544893-01	.37423148-01
.21104735-02	.20884360-01
-.37521483-01	-.99938655-01
.00000000	.00000000
-.37367455-01	-.11115301+00
.00000000	.00000000

.00000000	.00000000
-.15849355-01	-.63306928+00
-.62748775-02	-.23332037+00
-.66735897-02	-.14326847+00
.00000000	.00000000
-.83438142-02	-.11576757+00
-.12767664-01	-.84872157-01
-.12767664-01	-.84872157-01
-.44317652-01	-.28222599+00
.00000000	.00000000
-.33072399-01	-.17771619+00
.21104735-02	.20884360-01
.25544893-01	.37423148-01
-.68531631-01	-.26408821+00
.00000000	.00000000
-.37367455-01	-.11115301+00

COMPLEX LIFT DISTRIBUTION FOR MODE 5

.30151134+01	-.22613350+00	.13400504+01	-.10050378+00	.33501260+00	-.25125945-01	.00000000	.00000000
.33501260+00	-.25125945-01	.13400504+01	-.10050378+00	.30151134+01	-.22613350+00	.18011220+01	-.17584136+00
.66631309+00	-.66553782-01	.98910287-01	-.11910189-01	.98910287-01	-.11910189-01	.66631309+00	-.66553782-01
.18011220+01	-.17584136+00	.00000000	.00000000	.10835008+01	-.14326753+00	.37073694+00	-.48049173-01
.13314146+00	-.16310091-01	.37073694+00	-.48049173-01	.10835008+01	-.14326753+00	.00000000	.00000000
-.23491139+00	-.22839796-01	.67331652+00	-.12303138+00	.30397794+00	-.48581552-01	.30397794+00	-.48581552-01
.67331652+00	-.12303138+00	-.23491139+00	-.22839796-01	.00000000	.00000000	-.31983645+00	-.31048054-01
.55063932+00	-.13215728+00	.43213132+00	-.88405914-01	.55063932+00	-.13215728+00	-.31983645+00	-.31048054-01
.00000000	.00000000	-.79449140+00	.64573897-01	-.19948903+00	-.53964871-01	.64870027+00	-.18055748+00
.64870027+00	-.18055748+00	-.19948903+00	-.53964871-01	-.79449140+00	.64573897-01	.00000000	.00000000
-.92118597+00	.59936732-01	.65519203-01	-.11229653+00	.84138510+00	-.28328375+00	.65519203-01	-.11229653+00
-.92118597+00	.59936732-01	.00000000	.00000000	-.63000216+00	.19244406-01	-.71156360+00	.29574433-02
.39907994+00	-.22667570+00	.39907994+00	-.22667570+00	-.71156360+00	.29574433-02	-.63000216+00	.19244406-01
.00000000	.00000000	-.70630998+00	-.16989932-01	-.34706538+00	-.11004520+00	.19071667+00	-.17937019+00
-.34706538+00	-.11004520+00	-.70630998+00	-.16989932-01	.00000000	.00000000	.00000000	.00000000
-.43771360+00	-.99570740-01	-.51758538+00	-.11391760+00	-.39462838+00	-.58287018-01	-.39462838+00	-.58287018-01
-.51758538+00	-.11391760+00	-.43771360+00	-.99570740-01	.00000000	.00000000		

COMPLEX LIFT DISTRIBUTION FOR MODE 6

.00000000	.90453402+00	.00000000	.60302268+00	.00000000	.30151134+00	.00000000	.00000000
.00000000	-.30151134+00	.00000000	-.60302268+00	.00000000	-.90453402+00	.17952136-01	.74952886+00
.10771257-01	-.44971674+00	.35904240-02	.14990563+00	-.35904240-02	-.14990563+00	-.10771257-01	-.44971674+00
-.17952136-01	-.74952886+00	.00000000	.00000000	.27876889-01	.59785292+00	.13938148-01	.29892918+00
-.57683290-06	.58624287-05	-.13938148-01	-.29892918+00	-.27876889-01	-.59785292+00	.00000000	.00000000
.16509941-01	.13904035+00	.32861441-01	.44753855+00	.10953362-01	.14918432+00	-.10953362-01	-.14918432+00
-.32861441-01	-.44753855+00	-.16509941-01	-.13904035+00	.00000000	.00000000	.20033396-01	.15300377+00
.31659687-01	.29718416+00	-.94699883-06	.15380523-04	-.31659687-01	-.29718416+00	-.20033396-01	-.15300377+00
.00000000	.00000000	-.14868622-01	-.68935837-01	.12507289-01	.10409088+00	.20702444-01	.14771063+00
-.20702444-01	-.14771063+00	-.12507289-01	-.10409088+00	.14868622-01	.68935837-01	.00000000	.00000000
-.30619228-01	-.12908347+00	-.38829450-02	.23799302-01	-.72818646-06	.34988607-04	.38829450-02	-.23799302-01
.30619228-01	.12908347+00	.00000000	.00000000	-.42043543-01	-.85745417-01	-.53678863-01	-.19316978+00
-.28883873-01	-.73415785-01	.28883873-01	.73415785-01	.53678863-01	.19316978+00	.42043543-01	.85745417-01
.00000000	.00000000	-.82757266-01	-.18488846+00	-.85969985-01	-.26445023+00	.18158135-05	.79038200-04
.85969985-01	.26445023+00	.82757266-01	.18488846+00	.00000000	.00000000	.00000000	.00000000
-.67616636-01	-.99574757-01	-.12721399+00	-.28368038+00	-.62084774-01	-.19268837+00	.62084774-01	.19268837+00
.12721399+00	.28368038+00	.67616636-01	.99574757-01	.00000000	.00000000		

COMPLEX LIFT DISTRIBUTION FOR MODE 7

.30151134+01	-.45226701+00	.20100756+01	-.30151134+00	.10050378+01	-.15075567+00	.00000000	.00000000
-.10050378+01	.15075567+00	-.20100756+01	.30151134+00	-.30151134+01	.45226701+00	.24884888+01	-.37980719+00
.14930914+01	-.22788395+00	.49769729+00	-.75961357-01	-.49769729+00	.75961357-01	-.14930914+01	.22788395+00
-.24884888+01	.37980719+00	.00000000	.00000000	.19762143+01	-.30812347+00	.98811636+00	-.15406199+00
.19034110-04	-.76401353-06	-.98811636+00	.15406199+00	-.19762143+01	.30812347+00	.00000000	.00000000
.45207992+00	-.34863464-01	.14725427+01	-.23569830+00	.49086379+00	-.78566816-01	-.49086379+00	.78566816-01
-.14725427+01	.23569830+00	-.45207992+00	.34863464-01	.00000000	.00000000	.49565184+00	-.54524796-02
.97278393+00	-.16016270+00	.51790943-04	-.39156999-05	-.97278393+00	.16016270+00	-.49565184+00	.54524796-02
.00000000	.00000000	-.22876804+00	.19433918+00	.33615944+00	.53590059-01	.48111070+00	-.81275273-01
-.48111070+00	.81275273-01	-.33615944+00	-.53590059-01	.22876804+00	-.19433918+00	.00000000	.00000000
-.42388238+00	.31994517+00	.77476658-01	.12516268+00	.11717925-03	-.13688884-04	-.77476658-01	-.12516268+00
.42388238+00	-.31994517+00	.00000000	.00000000	-.27629226+00	.29576372+00	-.62710160+00	.42816468+00
-.23360706+00	.20192751+00	.23360706+00	-.20192751+00	.62710160+00	-.42816468+00	.27629226+00	-.29576372+00
.00000000	.00000000	-.59288210+00	.51302385+00	-.84995048+00	.53152979+00	.26314563-03	-.42291218-04
.84995048+00	-.53152979+00	.59288210+00	-.51302385+00	.00000000	.00000000	.00000000	.00000000
-.31815078+00	.37788676+00	-.90524544+00	.69231078+00	-.62067198+00	.35564822+00	.62067198+00	-.35564822+00
.90524544+00	-.69231078+00	.31815078+00	-.37788676+00	.00000000	.00000000		

RECTANGULAR A=2., H=1.2, K=.3. MODES= 1, X, X*X, Y*Y, X*X*Y*Y, Y, XY

I	J = 1	J = 2	J = 3	J = 4	J = 5	J = 6	J = 7
1	9.5998-02	3.6318+00	-4.3514-01	2.6097-02	-2.7130-01	2.1711-08	-1.5847-05
2	5.4724-03	-4.2984-01	9.3045-01	1.9174-03	2.4287-01	-2.6521-08	-3.7191-06
3	5.3576-03	3.0294-01	-4.8141-02	1.7646-03	-3.7968-02	2.9009-08	1.2741-05
4	1.6777-02	8.1932-01	-1.0470-01	1.0707-03	-6.2895-02	-1.0419-06	5.5471-05
5	9.6168-04	8.1611-02	-2.5772-02	5.5563-05	-2.2787-02	-6.5191-07	3.6941-05
6	0.0000	1.7462-10	0.0000	2.3647-11	0.0000	-1.0177-02	4.4197-01
7	8.9008-08	-4.6569-06	1.7196-05	2.2620-08	1.2081-05	-7.4297-03	-2.2834-01

I	J = 1	J = 2	J = 3	J = 4	J = 5	J = 6	J = 7
1	3.6882+00	-1.2953+00	1.5341+00	9.8922-01	4.6454-01	-1.5805-05	7.0158-06
2	-4.2644-01	4.0463-01	-4.0870-02	-1.4436-01	-2.6090-03	-3.7239-06	2.0031-06
3	3.0615-01	-8.4052-02	1.1583-01	9.3826-02	4.1375-02	1.2700-05	-1.0387-05
4	8.3055-01	-2.4088-01	3.2446-01	2.8794-01	9.6314-02	5.5149-05	-1.8630-05
5	8.2302-02	-2.3662-02	2.9860-02	3.9126-02	1.3574-02	3.6696-05	-1.4668-05
6	-3.8805-10	-4.8506-10	4.8506-10	-3.2742-10	1.4552-10	4.4201-01	1.5184-01
7	-4.5588-06	7.5979-06	-2.1935-07	-7.3298-06	-1.2879-06	-2.3087-01	2.2136-01

@FIN

Appendix A
SUBROUTINE DESCRIPTIONS

Appendix A

This appendix contains a brief outline on the purpose, method and use of each of the eight subroutines. The principal input and output variables are described. The subroutines are arranged in alphabetical order as follows:

KERNEL
LGSPAN
POLYGN
QIJ
SGRHBS
SGTRGL
SINGUL
TABLE

SUBROUTINE KERNEL

PURPOSE: To evaluate the reduced kernel function $\overline{K} = y_o^2 K/2$, where K is the supersonic kernel function.

METHOD: The nonplanar form of the oscillatory supersonic kernel function was derived by Harder and Rodden and was reduced to the planar form by A.M. Cunningham in the appendix, J. Aircraft, Vol. 11, No. 10, October 1974, pp.615.

USE: CALL KERNEL (XO, YO, XKREAL, XKIMAG)

Input

XO, YO Coordinates of the point of evaluation relative to the collocation point.

Output:

XKREAL, XKIMAG Real and imaginary parts of the reduced kernel function.

SUBROUTINE LGSPAN

PURPOSE: To integrate a function with inverse square singularity over half of the singular strip by extracting the Cauchy's principal value.

METHOD: Since a singular element contains either the complete strip or half of the strip, it is more convenient to treat only half a strip at a time. As the inverse square singularity occurs only in the spanwise integration, the chordwise integration can be performed first as

$$F(\eta) = \int_{\xi_a(\eta)}^{\xi_b(\eta)} f(\xi, \eta)\, d\xi$$

$$= \frac{\xi_b(\eta) - \xi_a(\eta)}{2} \sum_i^n w_i\, f_i$$

with

$$f_i = f(\xi_i, \eta)$$

and

$$\xi_i = \frac{\xi_b(\eta) - \xi_a(\eta)}{2} \zeta_i + \frac{\xi_b(\eta) + \xi_a(\eta)}{2}$$

where w_i and ζ_i are some Gaussian weights and abscissas over (-1, 1).

A sixth degree quadrature based on Lagrangian interpolation in conjunction with the Cauchy's principal value was devised by Watkins as

$$\neq\!\!\!\!\int_{y-\epsilon}^{y+\epsilon} \frac{F(\eta)}{(y-\eta)^2} dy$$

$$= \frac{1}{100\epsilon}\Big[13(F_1 + F_7) + 72(F_2 + F_6)$$

$$+ 495(F_3 + F_5) + (-1360)\, F_4\Big]$$

For use in this subroutine, the above quadrature is modified to

$$\oint_{y-\epsilon}^{y} \frac{F(\eta)}{(y-\eta)^2}\, dy = \frac{1}{100\epsilon}\left[13F_1 + 72F_2 + 496F_3 - 680F_4\right]$$

for the left half, and to

$$\oint_{y}^{y+\epsilon} \frac{F(\eta)}{(y-\eta)^2}\, dy = \frac{1}{100\epsilon}\left[13F_7 + 72F_6 + 495F_5 - 680F_4\right]$$

for the right half.

USE: CALL LGSPAN(X, Y, XEL, YEL, EINT, W, MAXINT, F)

Input:

X,Y	Coordinates of the collocation point.
XEL, YEL	Arrays of nodal coordinates of the half strip.
EINT	Array of Gauss-Chebyshev quadrature abscissas.
W	Array of Gauss-Chebyshev quadrature weights.
MAXINT	Number of integration points.

Output:

F	Array of integrated values at nodal points.

SUBROUTINES CALLED: KERNEL

ERROR RETURNS: None

SUBROUTINE POLYGN

PURPOSE: To perform the integration over a regular triangular or quadrilateral element.

METHOD: The arbitrary triangular or quadrilateral element is mapped into a square region $-1 \leq \xi \leq 1$ and $-1 \leq \eta \leq 1$. Integration is accomplished by a repeated application of the Gauss-Chebyshev quadrature in both directions. Since the mapping is different for a triangle and a quadrilateral, different computational routines are employed.

USE: CALL POLYGN(X, Y, XEL, YEL, EINT, W, MAXINT, F, IANGLE)

Input:

X, Y	Coordinates of the collocation point.
XEL, YEL	Arrays of nodal coordinates of the element.
EINT	Array of Gauss-Chebyshev quadrature abscissas.
W	Array of Gauss-Chebyshev quadrature weights.
MAXINT	Number of integration points.
IANGLE	Number of angles in the polygon element.

Output:

F	Array of integrated values at nodal points.

SUBROUTINE CALLED: KERNEL

ERROR RETURNS: None

SUBROUTINE QIJ

PURPOSE: To compute the generalized force coefficients for a given load distribution.

METHOD: With the finite element approximation, the generalized force coefficients are

$$Q_{ij} = -\sum_{n=1}^{N} \iint_{A^{(e)}} f_i(x, y)\, \overline{N}^t(x, y)\, dx\, dy \cdot \overline{\lambda}_j^{(e)}$$

where $\sum_{n=1}^{N}$ denotes summation over all elements,

f_i are the modes, $\overline{N}^t$ are the shape functions and $\overline{\lambda}_j^{(e)}$ is the elemental lift vector. The real and imaginary parts are defined as

$$Q'_{ij} = \mathrm{Re}(Q_{ij})$$

and

$$Q''_{ij} = \mathrm{Im}(Q_{ij})/k$$

where k is the reduced frequency. The integrations are performed using the Gaussian quadrature (2 points as set up in the subroutine).

USE: CALL QIJ(X, Y, INFO, XLIFT, FF, QREAL, QIMAG, NP, NELEM, NMODE, MX)

Input:

NP	Number of nodal points.
NELEM	Number of elements.
NMODE	Number of modes.
MX	Maximum number of nodes the the elements.
X, Y	Arrays of nodal coordinates.
INFO	Array of element information.
XLIFT	Array of modal lift distribution.
FF	Array for temporary storage.

Output:

QREAL	Real part of the generalized force coefficient, Q'_{ij}.
QIMAG	Imaginary part of the generalized force coefficient, Q''_{ij}.

SUBROUTINES CALLED: None

ERROR RETURNS: None

SUBROUTINE SGRHBS

PURPOSE: To perform the integration of a rhombic element with the singular strip passing through either the left, middle or right node.

METHOD: The rhombic element is divided into the right half triangle and the left half triangle. These triangular elements can be either regular or singular. The integrations are accomplished by calling another subroutine and the results are re-assembled.

USE:

```
CALL SGRHBS(XNODE, YNODE, XEL, YEL, EINT,
       W, MAXINT, F, DEL)
```

<u>Input:</u>

XNODE, YNODE	Coordinates of the collocation point.
XEL, YEL	Arrays of nodal coordinates of the element.
EINT	Array of Gauss-Chebyshev quadrature abscissas.
W	Array of Gauss-Chebyshev quadrature weights.
MAXINT	Number of integration points.
DEL	Ratio of the singular strip half width to the element half width.

<u>Output:</u>

F	Array of integrated values at nodal points.

SUBROUTINES CALLED: SINGUL

ERROR RETURNS: None

SUBROUTINE SGTRGL

PURPOSE:	To integrate a general triangular element with the singular strip passing through either one of the three vertices.
METHOD:	A vertical line through the lower vertex divides the element into two sub-triangles, which can be either regular or singular. The integrations are accomplished by calling another subroutine and the results are re-assembled.
USE:	CALL SGTRGL(XNODE, YNODE, XEL, YEL, EINT, W, MAXINT, F, DEL)

Input:

XNODE, YNODE	Coordinates of the collocation point.
XEL, YEL	Arrays of nodal coordinates of the element.
EINT	Array of Gauss-Chebyshev quadrature abscissas.
W	Array of Gauss-Chebyshev quadrature weights.
MAXINT	Number of integration points.
DEL	Ratio of the singular strip half width to the element half width.

Output:

F	Array of integrated values at nodal points.

SUBROUTINES CALLED:	SINGUL
ERROR RETURNS:	None

SUBROUTINE SINGUL

PURPOSE:	To integrate the special type of triangular element bounded by a vertical line and two other straight lines. The triangular element may be singular or regular.
METHOD:	The triangular element is first tested to see if it is singular or regular. For singular element, further test is conducted to locate the singularity with respect to the element such that the element can be divided into a singular strip and a regular polygon. Integrations are accomplished by calling other subroutines and the results are re-assembled.
USE:	CALL SINGUL(XNODE, YNODE, XEL, YEL, EINT, W, MAXINT, F, DEL)

<u>Input:</u>

XNODE, YNODE	Coordinates of the collocation point.
XEL, YEL	Arrays of nodal coordinates of the element.
EINT	Array of Gauss-Chebyshev quadrature abscissas.
W	Array of Gauss-Chebyshev quadrature weights.
MAXINT	Number of integration points.
DEL	Ratio of the singular strip half width to the element half width.

<u>Output:</u>

F	Array of integrated values at nodal points.

SUBROUTINES CALLED:	POLYGN, LGSPAN
ERROR RETURNS:	None

SUBROUTINE TABLE

PURPOSE:	To create a table of the weighted kernel function coefficients for a given uniform characteristic mesh. This table is stored for later table look-up in the solution process.
METHOD:	A stencil of uniform characteristic mesh large enough to cover the most extreme case for the planform is set up. Because of the symmetry in the spanwise direction, only the rhombic elements on one side need to be evaluated and stored. Each element is uniquely defined by a pair of relative indices based on its relative location from the collocation point.
USE:	CALL TABLE(COEF, DELTA, EINT, W, MAXINT, DEL, IMAX, JMAX, LMAX)

__Input:__

DELTA	Length of the side of the characteristic element.
EINT	Array of Gauss-Chebyshev quadrature abscissas.
W	Array of Gauss-Chebyshev quadrature weights.
MAXINT	Number of integration points.
DEL	Ratio of the singular strip half width to the element half width.
IMAX	Number of elements in the chordwise direction for the stencil.
JMAX	Number of elements in the Mach line direction for the stencil.
LMAX	Number of entries to the table.

__Output:__

COEF	Array of the weighted kernel function coefficients

SUBROUTINES CALLED:	SGRHBS, POLYGN
ERROR RETURNS:	None

Appendix B
PROGRAM LISTING OF FESKAP

```
      PROGRAM FESKAP(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C A NEW FINITE ELEMENT KERNEL FUNCTION METHOD IN SUPERSONIC
C LIFTING SURFACE THEORY
C DEVELOPED BY V. Y. C. YOUNG AT LOCKHEED - HUNTSVILLE
C
      COMPLEX F,SUM,UPWASH,CCEF,XLIFT
      DIMENSION X(67),Y(67),INFO(65,4),XLIFT(67,7),COEF(4,26),
     1          LENODE(7),IBUF(65),FF(67,7)
      DIMENSION F(4),XEL(4),YEL(4),SUM(7),UPWASH(7),TITLE(8),
     1          GREAL(7,7),GIMAG(7,7),W(30),EINT(30)
      COMMON XM,XK,BETASQ,BETA
      EQUIVALENCE (FF(1,1),COEF(1,1))
C
C THE FIRST DIMENSION STATEMENT IS DIMENSIONED AS X(NP),Y(NP),
C INFO(NELEM,4),XLIFT(NP,NMODE),COEF(4,LMAX),LENODE(NLE),IBUF(NELEM),
C FF(NP,NMODE)
C CAN BE ALTERED BY THE USER TO FIT THE PROBLEM
C
C IBUF AND FF ARE TEMPORARY STORAGE ARRAYS
C
C DEL    = WIDTH OF THE SINGULAR STRIP RELATIVE TO THE WIDTH OF
C          THE ELEMENT
C DELTA  = LENGTH OF THE SIDE OF THE CHARACTERISTIC ELEMENT
C ICHECK = DUMMY RUN TO CHECK THE MESH CORRECTNESS
C          IF ICHECK IS SET TO 1
C          OTHERWISE ICHECK IS SET TO 0
C IMAX   = MAXIMUM NUMBER OF CHARACTERISTIC ELEMENT IN THE
C          CHORDWISE DIRECTION
C JMAX   = MAXIMUM NUMBER OF CHARACTERISTIC ELEMENT IN THE
C          MACH LINE DIRECTION
C LENODE = ARRAY CONTAINING THE LEADING EDGE NODES
C MAXINT = NUMBER OF INTEGRATION POINTS FOR THE MODIFIED
C          GAUSS-CHEBYSHEV QUADRATURE (MAXIMUM SET AT 30)
C LMAX   = IMAX*JMAX-(IMAX*(IMAX-1))/2
C NELEM  = NO. OF ELEMENTS
C NLE    = NO. OF LEADING EDGE NODES
C NMODE  = NO. OF MODE SHAPES
C NP     = NO. OF NODES
C XK     = REDUCED FREQUENCY
C XLAMDA = SWEPTBACK ANGLE OF LEADING EDGE IN DEGREES
C XM     = MACH NUMBER
C
      DATA TOL/1.E-5/
      NAMELIST /LIST1/XM,XK,DELTA,MAXINT,DEL,IMAX,JMAX,XLAMDA,NMODE,NP,
     1               NELEM,ICHECK,NLE,LENODE,X,Y
      INDREL(A,B)=IFIX(CST1*(A+BETA*B)+.5)+1
      LINEAR(I,J)=((I-1)*(JMAX+JMAX-I))/2+J
C
C READ TITLE CARD AND NAMELIST INPUT
C
      READ(5,9200) TITLE
      READ(5,LIST1)
      BETASQ=XM*XM-1.
      BETA=SQRT(BETASQ)
      CST1=.5*XM/(BETA*DELTA)
      IF (ICHECK.EQ.1) WRITE(6,9610) TITLE
      IF (ICHECK.EQ.1) WRITE(6,9210) (I,X(I),Y(I),I=1,NP)
```

```
C
C GENERATE ELEMENT INFORMATION FROM DATA CARDS
C
      READ(5,9000) KMAX
      IF (ICHECK.EQ.1) WRITE(6,9620)
      DO 12 K=1,KMAX
      READ(5,9000) IREPT,I1,N1,N2,N3,N4
      DO 10 I=1,IREPT
      IM1=I-1
      I2=I1+IM1
      INFO(I2,1)=N1+IM1
      INFO(I2,2)=N2+IM1
      INFO(I2,3)=N3+IM1
      INFO(I2,4)=N4+IM1
      IF (N4.EQ.0) INFO(I2,4)=0
      IF (ICHECK.EQ.1) WRITE(6,9600) I2,INFO(I2,1),INFO(I2,2),
     1                               INFO(I2,3),INFO(I2,4)
   10 CONTINUE
   12 CONTINUE
      IF (ICHECK.EQ.1) GO TO 5001
C
C GENERATE THE MODIFIED GAUSS-CHEBYSHEV WEIGHTS AND ABSCISSAS
C
      CST=3.14159265359/MAXINT
      DO 20 I=1,MAXINT
      ARG=(I-.5)*CST
      EINT(I)=COS(ARG)
   20 W(I)=CST*SIN(ARG)
C
C CLEAR THE NODAL LIFT VALUES
C
      DO 30 MODE=1,NMODE
      DO 30 I=1,NP
   30 XLIFT(I,MODE)=CMPLX(0.,0.)
C
C GENERATE THE WEIGHTED KERNEL FUNCTION COEFFICIENTS FOR THE STENCIL
C OF UNIFORM CHARACTERISTIC MESH
C
      LMAX=LINEAR(IMAX,JMAX)
      CALL TABLE(COEF,DELTA,EINT,W,MAXINT,DEL,IMAX,JMAX,LMAX)
C
C COMPUTE THE SWEEPBACK FACTOR
C FOR SUBSONIC LEADING EDGE, SET SWEEPBACK FACTOR TO 1.
C
      CST=TAN(.174532925E-1*XLAMDA)/BETA
      SWPBK=1.
      CST=1.-CST*CST
      IF (CST.GT.1.E-10) SWPBK=1./SQRT(CST)
      CST=-2.*SWPBK/BETA
C
C COMPUTE THE STARTING SOLUTION AT THE LEADING EDGE
C
      DO 40 I=1,NLE
      I1=LENODE(I)
      XNODE=X(I1)
      YNODE=Y(I1)
      XISQ=XNODE*XNODE
      YISQ=YNODE*YNODE
```

```
      XLIFT(I1,1)=CST*CMPLX(0.,XK)
      XLIFT(I1,2)=CST*CMPLX(1.,XK*XNODE)
      XLIFT(I1,3)=CST*CMPLX(XNODE+XNODE,XK*XISQ)
      XLIFT(I1,4)=CST*CMPLX(0.,XK*YISQ)
      XLIFT(I1,5)=CST*CMPLX(2.*XNODE*YISQ,XK*XISQ*YISQ)
      XLIFT(I1,NMODE-1)=CST*CMPLX(0.,XK*YNODE)
      XLIFT(I1,NMODE)=CST*CMPLX(YNODE,XK*XNODE*YNODE)
   40 CONTINUE
 5001 CONTINUE
C
C LOOP THROUGH THE UNKNOWN NODAL POINTS TO COMPUTE THE LIFT
C
      READ(5,9000) LMAX
      IF (ICHECK.EQ.1) WRITE(6,9910)
      DO 1000 LNODE=1,LMAX
      READ(5,9000) N1,N2,N3,IREPT
      DO 1000 I=1,IREPT
      IM1=I-1
      NODE=N1+IM1
      NEL=N2+IM1
      MIRROR=N3-IM1
      XNODE=X(NODE)
      YNODE=Y(NODE)
      IF (ICHECK.EQ.1) GO TO 5002
C COMPUTE THE MODAL UPWASH AT THE COLLOCATION POINT
      XISQ=XNODE*XNODE
      YISQ=YNODE*YNODE
      UPWASH(1)=CMPLX(0.,XK)
      UPWASH(2)=CMPLX(1.,XK*XNODE)
      UPWASH(3)=CMPLX(XNODE+XNODE,XK*XISQ)
      UPWASH(4)=CMPLX(0.,XK*YISQ)
      UPWASH(5)=CMPLX(2.*XNODE*YISQ,XK*XISQ*YISQ)
      UPWASH(NMODE-1)=CMPLX(0.,XK*YNODE)
      UPWASH(NMODE)=CMPLX(YNODE,XK*XNODE*YNODE)
C
C CLEAR THE SUMS BEFORE ACCUMULATION
C
      DO 100 MODE=1,NMODE
      SUM(MODE)=CMPLX(0.,0.)
  100 CONTINUE
 5002 CONTINUE
C
C LOOP THROUGH THE FORWARD ELEMENTS
C
      KOUNT=0
      DO 800 L=1,NEL
      IANGLE=4
      IF (INFO(L,4).EQ.0) IANGLE=3
C
C ASSIGN THE ELEMENT NODAL POSITION
C
      DO 200 K=1,IANGLE
      KK=INFO(L,K)
      XEL(K)=X(KK)
  200 YEL(K)=Y(KK)
      X0=XNODE-XEL(1)
      Y0=YNODE-YEL(1)
C
```

```
C SKIP THE TEST FOR THE PIVOTAL ELEMENT
C
      IF (L.EQ.NEL) GO TO 201
C
C SKIP THE ELEMENT IF IT IS NOT WITHIN THE FORWARD MACH CONE
C
      IF ((XO-BETA*ABS(YO)).LT.(-TOL)) GO TO 800
  201 CONTINUE
      KOUNT=KOUNT+1
      IBUF(KOUNT)=L
      IF (ICHECK.EQ.1) GO TO 800
      IF (IANGLE.EQ.3) GO TO 400
C
C REGULAR CHARACTERISTIC ELEMENTS
C COMPUTE THE RELATIVE INDICES
C
      LL=INDREL(XO,YO)
      MM=INDREL(XO,-YO)
      IF (LL.GT.MM) GO TO 300
C
C LOWER TRIANGLE OF TABLE
C
      L1=LINEAR(LL,MM)
      F(1)=COEF(1,L1)
      F(2)=COEF(2,L1)
      F(3)=COEF(3,L1)
      F(4)=COEF(4,L1)
      GO TO 600
  300 CONTINUE
C
C UPPER TRIANGLE OF TABLE
C
      L1=LINEAR(MM,LL)
      F(1)=COEF(1,L1)
      F(2)=COEF(4,L1)
      F(3)=COEF(3,L1)
      F(4)=COEF(2,L1)
      GO TO 600
C
C TRIANGULAR FILL-IN ELEMENTS
C
  400 CONTINUE
C
C TEST FOR SINGULARITY
C
      DO 500 K=1,IANGLE
      IF (ABS(YNODE-YEL(K)).LT.TOL) GO TO 502
  500 CONTINUE
C
C NON-SINGULAR POLYGON
C
      CALL POLYGN(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,IANGLE)
      GO TO 600
C
C SINGULAR TRIANGLE
C
  502 CALL SGTRGL(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,DEL)
  600 CONTINUE
```

```
      DO 700 K=1,IANGLE
      KK=INFO(L,K)
      DO 700 MODE=1,NMODE
      SUM(MODE)=SUM(MODE)+XLIFT(KK,MODE)*F(K)
  700 CONTINUE
  800 CONTINUE
      IF (ICHECK.NE.1) GO TO 5003
      WRITE(6,9920) NODE,NEL,MIRROR
      WRITE(6,9900) (IBUF(K),K=1,KOUNT)
      GO TO 1000
 5003 CONTINUE
C
C BECAUSE THE PIVOTAL ELEMENT WAS TREATED LAST
C F(1) CONTAINS THE COEFFICIENT FOR THE UNKNOWN PIVOTAL LIFT
C SUBSTITUTE IN LIFT FOR MIRROR IMAGE TAKING ADVANTAGE OF SYMMETRY OR
C ANTI-SYMMETRY
C
      DO 900 MODE=1,NMODE
      XLIFT(NODE,MODE)=(6.28318530718*UPWASH(MODE)-SUM(MODE))/F(1)
      XLIFT(MIRROR,MODE)=XLIFT(NODE,MODE)
  900 CONTINUE
      XLIFT(MIRROR,NMODE-1)=-XLIFT(MIRROR,NMODE-1)
      XLIFT(MIRROR,NMODE)=-XLIFT(MIRROR,NMODE)
 1000 CONTINUE
      IF (ICHECK.EQ.1) STOP
      DO 2000 MODE=1,NMODE
      WRITE(6,9100) MODE
      WRITE(6,9500) (XLIFT(I,MODE),I=1,NP)
 2000 CONTINUE
C
C COMPUTE THE GENERALIZED FORCE COEFFICIENTS, GIJ
C
      CALL GIJ(X,Y,INFO,XLIFT,FF,GREAL,GIMAG,NP,NELEM,NMODE,4)
      WRITE(6,9300) TITLE
      IF (NMODE.EQ.6) WRITE(6,9810)
      IF (NMODE.EQ.7) WRITE(6,9800)
      DO 3000 I=1,NMODE
      WRITE(6,9400) I,(GREAL(I,J),J=1,NMODE)
 3000 CONTINUE
      WRITE(6,9700)
      IF (NMODE.EQ.6) WRITE(6,9810)
      IF (NMODE.EQ.7) WRITE(6,9800)
      DO 3100 I=1,NMODE
      WRITE(6,9400) I,(GIMAG(I,J),J=1,NMODE)
 3100 CONTINUE
      STOP
 9000 FORMAT(6I5)
 9100 FORMAT(1H1,34HCOMPLEX LIFT DISTRIBUTION FOR MODE,I2,/)
 9200 FORMAT(8A10)
 9210 FORMAT(1X,I5,10X,2E15.5)
 9300 FORMAT(1H1,///,5X,8A10/////)
 9400 FORMAT(1H0,I5,1P7E15.4)
 9500 FORMAT(2E15.8,4X,2E15.8,4X,2E15.8,4X,2E15.8)
 9600 FORMAT(1X,I5,10X,4I5)
 9610 FORMAT(1H1,///,1X,8A10,////,1X,15HMESH CHECK.....,/////,2X,
     1       4HNODE,13X,10HX-ORDINATE,5X,10HY-ORDINATE,//)
 9620 FORMAT(/////,1X,7HELEMENT,9X,5HNODES,//)
 9700 FORMAT(////)
```

```
9800 FORMAT(5X,1HI,8X,5HJ = 1,10X,5HJ = 2,10X,5HJ = 3,10X,5HJ = 4,
     1         10X,5HJ = 5,10X,5HJ = 6,10X,5HJ = 7/)
9810 FORMAT(5X,1HI,8X,5HJ = 1,10X,5HJ = 2,10X,5HJ = 3,10X,5HJ = 4,
     1         10X,5HJ = 5,10X,5HJ = 6/)
9900 FORMAT(32X,25I4)
9910 FORMAT(/////////,3X,4HNODE,6X,3HNEL,4X,6HMIRROR,6X,
     1       20HINFLUENCING ELEMENTS,//)
9920 FORMAT(1X,I5,2I10)
     END
```

```
      SUBROUTINE KERNEL(XO,YO,XKREAL,XKIMAG)
C
C SUPERSONIC KERNEL FUNCTION DERIVED BY HARDER AND RODDEN
C AS GIVEN BY A M CUNNINGHAM IN APPENDIX OF J. AIRCRAFT.
C VOL. 11. NO. 10. 1974.
C
      DIMENSION A(11)
      COMMON XM,XK,BETASQ,BETA
      DATA A/-.24186198,2.7968027,-24.991079,111.59196,-271.43549,
     1        305.75288,41.163630,-545.98537,644.78155,-328.72755,
     2        64.279511/
      DATA C/.372/
      R=SQRT(XO*XO-BETASQ*YO*YO)
      IF (XK.LT.1.E-5) GO TO 400
      AYO=ABS(YO)
      ARG=XK*XO
      CS=COS(ARG)
      SN=SIN(ARG)
      IF (AYO.LT.1.E-5) GO TO 500
C
C GERNERAL FORM OF KERNEL FUNCTION
C
      XKAYO=XK*AYO
      XKYOSQ=XKAYO*XKAYO
      B2YO1=1./(BETASQ*AYO)
      XMR=XM*R
      U1=B2YO1*(XO-XMR)
      U2=B2YO1*(XO+XMR)
      E1=EXP(-C*ABS(U1))
      E2=EXP(-C*U2)
      CST=0.
      CST1=1.
      CST2=1.
      SUM=0.
      SUM1RL=0.
      SUM1IM=0.
      SUM2RL=0.
      SUM2IM=0.
      DO 100 I=1,11
      CST=CST+C
      CST1=CST1*E1
      CST2=CST2*E2
      COEF=A(I)/(CST*CST+XKYOSQ)
      SUM=SUM+COEF
      COEFRL=CST*COEF
      COEFIM=-XKAYO*COEF
      SUM1RL=SUM1RL+COEFRL*CST1
      SUM1IM=SUM1IM+COEFIM*CST1
      SUM2RL=SUM2RL+COEFRL*CST2
      SUM2IM=SUM2IM+COEFIM*CST2
  100 CONTINUE
      ARG=XKAYO*U1
      CS1=COS(ARG)
      SN1=SIN(ARG)
      ARG=XKAYO*U2
      CS2=COS(ARG)
      SN2=SIN(ARG)
      XI12RL=XKAYO*(SN2*SUM2RL-CS2*SUM2IM)
```

```
      XI12IM=XKAYO*(CS2*SUM2RL+SN2*SUM2IM)
      CST1=SN1*SUM1RL
      CST2=CS1*SUM1IM
      CST3=CS1*SUM1RL
      CST4=SN1*SUM1IM
      IF (U1.LT.0.) GO TO 200
      XI11RL=XKAYO*(CST1-CST2)
      XI11IM=XKAYO*(CST3+CST4)
      GO TO 300
  200 XI11RL=XKAYO*(CST1+CST2)+2.*(CS1-1.+XKYOSQ*SUM)
      XI11IM=XKAYO*(CST3-CST4)-SN1-SN1
  300 CONTINUE
      CST1=(XO/R)+1.
      CST2=CST1-2.
      XK11RL=CST1*CS1-XI11RL
      XK11IM=-CST1*SN1-XI11IM
      XK12RL=CST2*CS2+XI12RL
      XK12IM=-CST2*SN2+XI12IM
      SUM1RL=XK11RL+XK12RL
      SUM1IM=XK11IM+XK12IM
      XKREAL=.5*(CS*SUM1RL+SN*SUM1IM)
      XKIMAG=.5*(CS*SUM1IM-SN*SUM1RL)
      RETURN
C
C STEADY FORM OF KERNEL FUNCTION
C
  400 XKREAL=XO/R
      XKIMAG=0.
      RETURN
C
C SPECIAL FORM OF KERNEL FUNCTION AT YO=0
C
  500 XKREAL=CS
      XKIMAG=-SN
      RETURN
      END
```

```
      SUBROUTINE LGSPAN(X,Y,XEL,YEL,EINT,W,MAXINT,F)
C
C INTEGRATION OVER HALF OF THE SINGULAR STRIP
C WITH CAUCHYS PRINCIPAL VALUE
C USING A SIXTH DEGREE LAGRANGIAN INTERPOLATION
C
      REAL N
      COMPLEX XKBAR,F,SUM
      DIMENSION F(4),SUM(4),N(4)
      DIMENSION COEF(4),S(4),XEL(4),YEL(4),EINT(MAXINT),W(MAXINT)
      DATA COEF/.13,.72,4.95,-6.8/
      DATA S/1.,.33333333,-.33333333,-1./
      SFCN(A,B)=.25*(1.+A)*(1.+B)
C
C TEST TO SEE IF SINGULARITY IS TO LEFT OR RIGHT
C
      IFLIP=0
      IF ((Y+Y-YEL(1)-YEL(2)).GT.0) IFLIP=1
      EPS=ABS(YEL(2)-YEL(1))
      DO 10 II=1,4
   10 F(II)=CMPLX(0.,0.)
C
C BEGIN SPANWISE INTEGRATION
C
      DO 1000 I=1,4
C
C COMPUTE THE UPPER AND LOWER LIMITS FOR THE CHORDWISE INTEGRATION
C REVERSE SIGN IF SINGULARITY IS TO THE RIGHT
C
      Z1=S(I)
      IF (IFLIP.EQ.1) Z1=-Z1
      CST1=.5*(1.-Z1)
      CST2=.5*(1.+Z1)
      A=CST1*XEL(4)+CST2*XEL(3)
      B=CST1*XEL(1)+CST2*XEL(2)
      C1=.5*(B-A)
      C2=.5*(B+A)
      Y0=Y-CST1*YEL(1)-CST2*YEL(2)
      DO 20 II=1,4
   20 SUM(II)=CMPLX(0.,0.)
C
C BEGIN CHORDWISE INTEGRATION
C
      DO 100 L=1,MAXINT
      Z2=EINT(L)
      X0=X-(C1*Z2+C2)
      CALL KERNEL(X0,Y0,XKREAL,XKIMAG)
      XKBAR=CMPLX(XKREAL,XKIMAG)
      N(1)=SFCN(-Z1,-Z2)
      N(2)=SFCN(Z1,-Z2)
      N(3)=SFCN(Z1,Z2)
      N(4)=SFCN(-Z1,Z2)
      DO 30 II=1,4
   30 SUM(II)=SUM(II)+W(L)*N(II)*XKBAR
  100 CONTINUE
C
C END CHORDWISE INTEGRATION
C
```

```
      CST=C1*COEF(I)
      DO 40 II=1,4
   40 F(II)=F(II)+CST*SUM(II)
 1000 CONTINUE
      DO 50 II=1,4
   50 F(II)=F(II)/EPS
C
C END SPANWISE INTEGRATION
C
      RETURN
      END
```

```
      SUBROUTINE POLYGN(X,Y,XEL,YEL,EINT,W,MAXINT,F,IANGLE)
C
C INTEGRATION OVER A REGULAR TRIANGULAR OR QUADRILATERAL ELEMENT
C AS INDICATED BY THE VALUE OF IANGLE
C
      REAL N
      COMPLEX XKBAR,F,SUM
      DIMENSION F(4),SUM(4),N(4),XEL(4),YEL(4)
      DIMENSION EINT(MAXINT),W(MAXINT)
      SFCN(A,B)=.25*(1.+A)*(1.+B)
      IFLAG=IANGLE-2
      DO 100 II=1,4
  100 F(II)=CMPLX(0.,0.)
      X31=XEL(3)-XEL(1)
      Y31=YEL(3)-YEL(1)
      GO TO (200,300),IFLAG
C
C TRIANGLE
C
  200 X23=XEL(2)-XEL(3)
      Y23=YEL(2)-YEL(3)
C
C JACOBIAN / 4
C
      XJCBN=.25*(Y31*X23-X31*Y23)
      DO 250 J=1,MAXINT
      XJ=EINT(J)
      N(2)=.5*(1.+XJ)
      C1=.5*(1.-XJ)
      WJ=C1*W(J)
      DO 210 II=1,3
  210 SUM(II)=CMPLX(0.,0.)
      DO 230 I=1,MAXINT
      XI=EINT(I)
      N(1)=.5*C1*(1.+XI)
      N(3)=C1-N(1)
      ZETA=N(1)*XEL(1)+N(2)*XEL(2)+N(3)*XEL(3)
      ETA=N(1)*YEL(1)+N(2)*YEL(2)+N(3)*YEL(3)
      X0=X-ZETA
      Y0=Y-ETA
      CALL KERNEL(X0,Y0,XKREAL,XKIMAG)
      XKBAR=CMPLX(XKREAL,XKIMAG)
      WI=W(I)/(Y0*Y0)
      DO 220 II=1,3
  220 SUM(II)=SUM(II)+WI*N(II)*XKBAR
  230 CONTINUE
      DO 240 II=1,3
  240 F(II)=F(II)+WJ*SUM(II)
  250 CONTINUE
      GO TO 400
C
C QUADRILATERAL
C
  300 X42=XEL(4)-XEL(2)
      Y42=YEL(4)-YEL(2)
C
C JACOBIAN / 8
C
```

```
      XJCBN=.125*(X31*Y42-Y31*X42)
      DO 350 I=1,MAXINT
      DO 310 K=1,4
  310 SUM(K)=CMPLX(0.,0.)
      Z1=EINT(I)
      DO 330 J=1,MAXINT
      Z2=EINT(J)
      N(1)=SFCN(-Z1,-Z2)
      N(2)=SFCN(Z1,-Z2)
      N(3)=SFCN(Z1,Z2)
      N(4)=SFCN(-Z1,Z2)
      ZETA=N(1)*XEL(1)+N(2)*XEL(2)+N(3)*XEL(3)+N(4)*XEL(4)
      ETA=N(1)*YEL(1)+N(2)*YEL(2)+N(3)*YEL(3)+N(4)*YEL(4)
      XO=X-ZETA
      YO=Y-ETA
      CALL KERNEL(XO,YO,XKREAL,XKIMAG)
      XKBAR=CMPLX(XKREAL,XKIMAG)
      WJ=W(J)/(YO*YO)
      DO 320 K=1,4
  320 SUM(K)=SUM(K)+WJ*N(K)*XKBAR
  330 CONTINUE
      DO 340 K=1,4
  340 F(K)=F(K)+W(I)*SUM(K)
  350 CONTINUE
  400 CONTINUE
      DO 500 K=1,4
  500 F(K)=XJCBN*F(K)
      RETURN
      END
```

```
      SUBROUTINE QIJ(X,Y,INFO,XLIFT,FF,QREAL,QIMAG,NP,NELEM,NMODE,MX)
C
C THIS SUBOUTINE COMPUTES THE GENERALIZED FORCE COEFFICIENTS
C FOR A GIVEN LOAD DISTRIBUTION
C
      REAL N
      COMPLEX XLIFT
      DIMENSION X(NP),Y(NP),INFO(NELEM,MX),XLIFT(NP,NMODE),FF(NP,NMODE)
      DIMENSION XEL(4),YEL(4),F(4,7),SUM(4,7),FI(7),N(4),EINT(2),W(2),
     1          QREAL(7,7),QIMAG(7,7)
      COMMON XM,XK,BETASQ,BETA
      DATA MAXINT/2/
      DATA EINT/.577350269189626,-.577350269189626/
      DATA W/2*1./
      SFCN(A,B)=.25*(1.+A)*(1.+B)
C
C INITIALIZE
C
      DO 2000 I=1,NMODE
      DO 2200 J=1,NMODE
      QREAL(I,J)=0.
 2200 QIMAG(I,J)=0.
      DO 2400 K=1,NP
 2400 FF(K,I)=0.
 2000 CONTINUE
C
C LOOP THROUGH THE ELEMENTS
C
      DO 4000 L=1,NELEM
      IANGLE=4
      IF (INFO(L,4).EQ.0) IANGLE=3
      DO 3000 K=1,IANGLE
      K1=INFO(L,K)
      XEL(K)=X(K1)
      YEL(K)=Y(K1)
 3000 CONTINUE
      IFLAG=IANGLE-2
C
C CLEAR F BEFORE ACCUMULATION
C
      DO 100 II=1,4
      DO 100 MODE=1,NMODE
  100 F(II,MODE)=0.
      X31=XEL(3)-XEL(1)
      Y31=YEL(3)-YEL(1)
C
C BEGIN INTEGRATION -- OUTER LOOP
C
      DO 250 J=1,MAXINT
      GO TO (251,252),IFLAG
C
C TRIANGULAR ELEMENT
C
  251 XJ=EINT(J)
      N(2)=.5*(1.+XJ)
      C1=.5*(1.-XJ)
      WJ=C1*W(J)
      GO TO 253
```

```
C
C QUADRILATERAL ELEMENT
C
  252 Z2=EINT(J)
      WJ=W(J)
  253 DO 210 II=1,4
      DO 210 MODE=1,NMODE
  210 SUM(II,MODE)=0.
C
C BEGIN INTEGRATION -- INNER LOOP
C
      DO 230 I=1,MAXINT
      GO TO (231,232),IFLAG
C
C TRIANGULAR ELEMENT
C
  231 XI=EINT(I)
      N(1)=.5*C1*(1.+XI)
      N(3)=C1-N(1)
      XP=N(1)*XEL(1)+N(2)*XEL(2)+N(3)*XEL(3)
      YP=N(1)*YEL(1)+N(2)*YEL(2)+N(3)*YEL(3)
      GO TO 233
C
C QUADRILATERAL ELEMENT
C
  232 Z1=EINT(I)
      N(1)=SFCN(-Z1,-Z2)
      N(2)=SFCN(Z1,-Z2)
      N(3)=SFCN(Z1,Z2)
      N(4)=SFCN(-Z1,Z2)
      XP=N(1)*XEL(1)+N(2)*XEL(2)+N(3)*XEL(3)+N(4)*XEL(4)
      YP=N(1)*YEL(1)+N(2)*YEL(2)+N(3)*YEL(3)+N(4)*YEL(4)
C
C COMPUTE THE MODAL DEFLECTION
C
  233 FI(1)=1.
      FI(2)=XP
      FI(3)=XP*XP
      FI(4)=YP*YP
      FI(5)=FI(3)*FI(4)
      FI(NMODE-1)=YP
      FI(NMODE)=XP*YP
      DO 220 II=1,IANGLE
      CST=W(I)*N(II)
      DO 220 MODE=1,NMODE
  220 SUM(II,MODE)=SUM(II,MODE)+CST*FI(MODE)
  230 CONTINUE
C
C END INNER LOOP
C
      DO 240 II=1,IANGLE
      DO 240 MODE=1,NMODE
  240 F(II,MODE)=F(II,MODE)+WJ*SUM(II,MODE)
  250 CONTINUE
C
C END OUTER LOOP
C
      GO TO (200,300),IFLAG
```

```
C
C TRIANGULAR ELEMENT
C
  200 X23=XEL(2)-XEL(3)
      Y23=YEL(2)-YEL(3)
      AREA=.25*(Y31*X23-X31*Y23)
      GO TO 400
C
C QUADRILATERAL ELEMENT
C
  300 X42=XEL(4)-XEL(2)
      Y42=YEL(4)-YEL(2)
      AREA=.125*(X31*Y42-Y31*X42)
C
C GLOBALIZE THE ELEMENTAL INTEGRATED COEFFICIENT VECTOR TO
C FORM THE INTEGRATED COEFFICIENT VECTOR
C
  400 DO 500 J=1,IANGLE
      K=INFO(L,J)
      DO 500 MODE=1,NMODE
  500 FF(K,MODE)=FF(K,MODE)+AREA*F(J,MODE)
 4000 CONTINUE
C
C COMPUTE THE GENERALIZED FORCE COEFFICIENTS Q(I,J)
C
      DO 5000 I=1,NMODE
      DO 5000 J=1,NMODE
      DO 5000 K=1,NP
      QREAL(I,J)=QREAL(I,J)-FF(K,I)*REAL(XLIFT(K,J))
      QIMAG(I,J)=QIMAG(I,J)-FF(K,I)*AIMAG(XLIFT(K,J))
 5000 CONTINUE
C
C DIVIDE IMAGINARY PART OF Q BY REDUCED FREQUENTCY
C SKIP THE CONVERSION FOR ZERO FREQUENCY
C
      IF (XK.LT.1.E-5) GO TO 9999
      DO 6000 I=1,NMODE
      DO 6000 J=1,NMODE
 6000 QIMAG(I,J)=QIMAG(I,J)/XK
 9999 RETURN
      END
```

```
      SUBROUTINE SGRHBS(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,DEL)
C
C THIS SUBROUTINE PERFORMS THE INTEGRATION OF A RHOMBIC ELEMENT
C WITH THE SINGULAR STRIP PASSING THROUGH EITHER THE LEFT,
C MIDDLE OR RIGHT NODE
C
      COMPLEX F,F1
      DIMENSION XEL(4),YEL(4),XEL1(4),YEL1(4),F(4),F1(4)
      DIMENSION EINT(MAXINT),W(MAXINT)
C
C INTEGRATE THE RIGHT HALF TRIANGULAR REGION
C
      CALL SINGUL(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,DEL)
C
C INTEGRATE THE LEFT HALF TRIANGULAR REGION
C
      XEL1(1)=XEL(1)
      YEL1(1)=YEL(1)
      XEL1(2)=XEL(3)
      YEL1(2)=YEL(3)
      XEL1(3)=XEL(4)
      YEL1(3)=YEL(4)
      CALL SINGUL(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1,DEL)
      F(1)=F(1)+F1(1)
      F(3)=F(3)+F1(2)
      F(4)=F1(3)
      RETURN
      END
```

```
      SUBROUTINE SGTRGL(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,DEL)
C
C INTEGRATION OVER A GENERAL TRIANGULAR ELEMENT WITH THE
C SINGULAR STRIP PASSING THROUGH EITHER ONE OF THE VERTICES
C
      COMPLEX F,F1
      DIMENSION XEL(4),YEL(4),XEL1(4),YEL1(4),F(4),F1(4)
      DIMENSION EINT(MAXINT),W(MAXINT)
      DEL1=(YEL(1)-YEL(3))/(YEL(2)-YEL(3))
      IF (DEL1.LT.1.E-5) GO TO 1000
      DM=1.-DEL1
      IF (DM.LT.1.E-5) GO TO 1000
C
C INTEGRATE THE LEFT SIDE OF THE TRIANGLE
C
      XEL1(1)=XEL(1)
      YEL1(1)=YEL(1)
      XEL1(2)=DEL1*XEL(2)+DM*XEL(3)
      YEL1(2)=DEL1*YEL(2)+DM*YEL(3)
      XEL1(3)=XEL(3)
      YEL1(3)=YEL(3)
      CALL SINGUL(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1,DEL)
      F(1)=F1(1)
      F(2)=DEL1*F1(2)
      F(3)=F1(3)+DM*F1(2)
C
C INTEGRATE THE RIGHT SIDE OF THE TRIANGLE
C
      XEL1(3)=XEL1(2)
      YEL1(3)=YEL1(2)
      XEL1(2)=XEL(2)
      YEL1(2)=YEL(2)
      CALL SINGUL(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1,DEL)
      F(1)=F(1)+F1(1)
      F(2)=F(2)+F1(2)+DEL1*F1(3)
      F(3)=F(3)+DM*F1(3)
      RETURN
C
C DEGENERATE CASE WHERE THE TRIANGULAR ELEMENT HAS ONE SIDE
C COINCIDE WITH THE SINGULAR STRIP
C
 1000 CALL SINGUL(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,DEL)
      RETURN
      END
```

```
      SUBROUTINE SINGUL(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,DEL)
C
C INTEGRATION OVER THE SPECIAL TYPE OF TRIANGULAR ELEMENT BOUNDED
C BY A VERTICAL LINE AND TWO OTHER STRAIGHT LINES
C THE TRIANGLE MAY BE SINGULAR OR REGULAR
C
      COMPLEX F,F1
      DIMENSION F(4),F1(4)
      DIMENSION XEL(4),YEL(4),XEL1(4),YEL1(4),EINT(MAXINT),W(MAXINT)
      H(A,B)=DEL*A+CM*B
      CM=1.-DEL
      IF (ABS(YEL(1)-YEL(2)).GT.1.E-5) GO TO 10
      IF (ABS(YNODE-YEL(3)).LT.1.E-5) GO TO 1
      IF (ABS(YNODE-YEL(2)).LT.1.E-5) GO TO 3
   10 CONTINUE
      IF (ABS(YNODE-YEL(2)).LT.1.E-5) GO TO 2
      IF (ABS(YNODE-YEL(3)).LT.1.E-5) GO TO 4
C
C NON-SINGULAR TRIANGLE
C
      CALL POLYGN(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,F,3)
      RETURN
C
C TRIANGLE POINTING LEFT WITH SINGULAR STRIP PASSING THROUGH
C THE VERTICAL SIDE
C
    1 XEL1(1)=XEL(3)
      YEL1(1)=YEL(3)
      XEL1(2)=H(XEL(1),XEL(3))
      YEL1(2)=H(YEL(1),YEL(3))
      XEL1(3)=H(XEL(2),XEL(3))
      YEL1(3)=H(YEL(2),YEL(3))
      XEL1(4)=XEL(3)
      YEL1(4)=YEL(3)
      CALL LGSPAN(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1)
      F(1)=DEL*F1(2)
      F(2)=DEL*F1(3)
      F(3)=F1(1)+F1(4)+CM*(F1(2)+F1(3))
      IF (ABS(1.-DEL).LT.1.E-5) RETURN
      XEL1(1)=XEL1(2)
      YEL1(1)=YEL1(2)
      XEL1(4)=XEL1(3)
      YEL1(4)=YEL1(3)
      XEL1(2)=XEL(1)
      YEL1(2)=YEL(1)
      XEL1(3)=XEL(2)
      YEL1(3)=YEL(2)
      CALL POLYGN(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1,4)
      F(1)=F(1)+F1(2)+DEL*F1(1)
      F(2)=F(2)+F1(3)+DEL*F1(4)
      F(3)=F(3)+CM*(F1(1)+F1(4))
      RETURN
C
C TRIANGLE POINTING RIGHT WITH SINGULAR STRIP PASSING THROUGH
C THE VERTICAL SIDE
C
    2 XEL1(1)=H(XEL(1),XEL(2))
      YEL1(1)=H(YEL(1),YEL(2))
```

```
      XEL1(2)=XEL(2)
      YEL1(2)=YEL(2)
      XEL1(3)=XEL(2)
      YEL1(3)=YEL(2)
      XEL1(4)=H(XEL(3),XEL(2))
      YEL1(4)=H(YEL(3),YEL(2))
      CALL LGSPAN(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1)
      F(1)=DEL*F1(1)
      F(2)=F1(2)+F1(3)+CM*(F1(1)+F1(4))
      F(3)=DEL*F1(4)
      IF (ABS(1.-DEL).LT.1.E-5) RETURN
      XEL1(2)=XEL1(1)
      YEL1(2)=YEL1(1)
      XEL1(3)=XEL1(4)
      YEL1(3)=YEL1(4)
      XEL1(1)=XEL(1)
      YEL1(1)=YEL(1)
      XEL1(4)=XEL(3)
      YEL1(4)=YEL(3)
      CALL POLYGN(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1,4)
      F(1)=F(1)+F1(1)+DEL*F1(2)
      F(2)=F(2)+CM*(F1(2)+F1(3))
      F(3)=F(3)+F1(4)+DEL*F1(3)
      RETURN
C
C TRIANGLE POINTING LEFT WITH SINGULAR STRIP PASSING THROUGH
C THE VERTEX OPPOSITE TO THE VERTICAL SIDE
C
    3 XEL1(1)=H(XEL(3),XEL(1))
      YEL1(1)=H(YEL(3),YEL(1))
      XEL1(2)=XEL(1)
      YEL1(2)=YEL(1)
      XEL1(3)=XEL(2)
      YEL1(3)=YEL(2)
      XEL1(4)=H(XEL(3),XEL(2))
      YEL1(4)=H(YEL(3),YEL(2))
      CALL LGSPAN(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1)
      F(1)=F1(2)+CM*F1(1)
      F(2)=F1(3)+CM*F1(4)
      F(3)=DEL*(F1(1)+F1(4))
      IF (ABS(1.-DEL).LT.1.E-5) RETURN
      XEL1(3)=XEL1(1)
      YEL1(3)=YEL1(1)
      XEL1(1)=XEL1(4)
      YEL1(1)=YEL1(4)
      XEL1(2)=XEL(3)
      YEL1(2)=YEL(3)
      CALL POLYGN(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1,3)
      F(1)=F(1)+CM*F1(3)
      F(2)=F(2)+CM*F1(1)
      F(3)=F(3)+F1(2)+DEL*(F1(1)+F1(3))
      RETURN
C
C TRIANGLE POINTING RIGHT WITH SINGULAR STRIP PASSING THROUGH
C THE VERTEX OPPOSITE TO THE VERTICAL SIDE
C
    4 XEL1(1)=XEL(1)
      YEL1(1)=YEL(1)
```

```
      XEL1(2)=H(XEL(2),XEL(1))
      YEL1(2)=H(YEL(2),YEL(1))
      XEL1(3)=H(XEL(2),XEL(3))
      YEL1(3)=H(YEL(2),YEL(3))
      XEL1(4)=XEL(3)
      YEL1(4)=YEL(3)
      CALL LGSPAN(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1)
      F(1)=F1(1)+CM*F1(2)
      F(2)=DEL*(F1(2)+F1(3))
      F(3)=F1(4)+CM*F1(3)
      IF (ABS(1.-DEL).LT.1.E-5) RETURN
      XEL1(1)=XEL1(2)
      YEL1(1)=YEL1(2)
      XEL1(2)=XEL(2)
      YEL1(2)=YEL(2)
      CALL POLYGN(XNODE,YNODE,XEL1,YEL1,EINT,W,MAXINT,F1,3)
      F(1)=F(1)+CM*F1(1)
      F(2)=F(2)+F1(2)+DEL*(F1(1)+F1(3))
      F(3)=F(3)+CM*F1(3)
      RETURN
      END
```

```
      SUBROUTINE TABLE(COEF,DELTA,EINT,W,MAXINT,DEL,IMAX,JMAX,LMAX)
C
C THIS SUBROUTINE CREATES A TABLE OF THE WEIGHTED KERNEL
C COEFFICIENTS FOR A REGULAR CHARACTERISTIC MESH
C EACH ELEMENT IS UNIQUELY DEFINED BY A PAIR OF RELATIVE INDICES
C DELTA IS THE LENGTH OF THE SIDE OF THE CHARACTERISTIC ELEMENT
C IMAX IS THE MAXIMUM POSSIBLE NUMBER OF ELEMENTS IN THE CHORDWISE DIREC
C JMAX IS THE MAXIMUM POSSIBLE NUMBER OF ELEMENTS IN THE MACH LINE DIREC
C LMAX IS THE MAXIMUM POSSIBLE ENTRIES TO THE TABLE
C THE TABLE IS COMPACTLY STORED INTO A RECTANGULAR ARRAY
C
      COMPLEX F,COEF
      DIMENSION F(4),COEF(4,LMAX)
      DIMENSION XEL(4),YEL(4),EINT(MAXINT),W(MAXINT)
      COMMON XM,XK,BETASQ,BETA
      LINEAR(I,J)=((I-1)*(JMAX+JMAX-I))/2+J
      YDEL=DELTA/XM
      XDEL=BETA*YDEL
      DO 20 LL=1,IMAX
      DO 20 MM=LL,JMAX
C
C DEFINE THE ELEMENT LOCATION ACCORDING TO ITS RELATIVE INDICES
C
      L=LINEAR(LL,MM)
      XEL(1)=(2-LL-MM)*XDEL
      XEL(2)=XEL(1)-XDEL
      XEL(3)=XEL(2)-XDEL
      XEL(4)=XEL(2)
      YEL(1)=(MM-LL)*YDEL
      YEL(2)=YEL(1)+YDEL
      YEL(3)=YEL(1)
      YEL(4)=YEL(1)-YDEL
C
C CHARACTERISTIC ELEMENT CUT BY THE SINGULAR STRIP
C
      IF ((MM-LL).LE.1) CALL SGRHBS(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,
     1                              F,DEL)
C
C REGULAR CHARACTERISTIC ELEMENT
C
      IF ((MM-LL).GT.1) CALL POLYGN(XNODE,YNODE,XEL,YEL,EINT,W,MAXINT,
     1                              F,4)
      DO 20 I=1,4
      COEF(I,L)=F(I)
   20 CONTINUE
      RETURN
      END
```


☆ U. S. GOVERNMENT PRINTING OFFICE: 1976 — 657-630/770

SUPPLEMENTARY

INFORMATION

AFFDL-TR-76-3
V2
Substitute pages

AD-B012040L

```
      SUBROUTINE KERNEL(XO,YO,XKREAL,XKIMAG)                          B 001
C                                                                     B 002
C SUPERSONIC KERNEL FUNCTION DERIVED BY HARDER AND RODDEN             B 003
C AS GIVEN BY A M CUNNINGHAM IN APPENDIX OF J. AIRCRAFT.              B 004
C VOL. 11. NO. 10. 1974. WITH CORRECTIONS AT LINES PRECEEDED BY **    B 005
C                                                                     B 006
      DIMENSION A(11)                                                 B 007
      COMMON XM,XK,BETASQ,BETA                                        B 008
      DATA A/-.24186198,2.7918027,-24.991079,111.59196,-271.43549,  **B 009
     1        305.75288,41.183530,-545.98537,644.78155,-328.72755,    B 010
     2        64.279511/                                              B 011
      DATA C/.372/                                                    B 012
      R=SQRT(XO*XO-BETASQ*YO*YO)                                      B 013
      IF (XK.LT.1.E-5) GO TO 400                                      B 014
      AYO=ABS(YO)                                                     B 015
      ARG=XK*XO                                                       B 016
      CS=COS(ARG)                                                     B 017
      SN=SIN(ARG)                                                     B 018
      IF (AYO.LT.1.E-5) GO TO 500                                     B 019
C                                                                     B 020
C GERNERAL FORM OF KERNEL FUNCTION                                    B 021
C                                                                     B 022
      XKAYO=XK*AYO                                                    B 023
      XKAYOSQ=XKAYO*XKAYO                                             B 024
      B2YO1=1./(BETASQ*AYO)                                           B 025
      XMR=XM*R                                                        B 026
      U1=B2YO1*(XO-XMR)                                               B 027
      U2=B2YO1*(XO+XMR)                                               B 028
      E1=EXP(-C*ABS(U1))                                              B 029
      E2=EXP(-C*U2)                                                   B 030
      CST=0.                                                          B 031
      CST1=1.                                                         B 032
      CST2=1.                                                         B 033
      SUM=0.                                                          B 034
      SUM1RL=0.                                                       B 035
      SUM1IM=0.                                                       B 036
      SUM2RL=0.                                                       B 037
      SUM2IM=0.                                                       B 038
      DO 100 I=1,11                                                   B 039
      CST=CST+C                                                       B 040
      CST1=CST1*E1                                                    B 041
      CST2=CST2*E2                                                    B 042
      COEF=A(I)/(CST*CST+XKAYOSQ)                                   **B 043
      SUM=SUM-COEF                                                    B 044
      COEFRL=CST*COEF                                                 B 045
      COEFIM=-XKAYO*COEF                                            **B 046
      SUM1RL=SUM1RL-COEFRL*CST1                                     **B 047
      SUM1IM=SUM1IM-COEFIM*CST1                                     **B 048
      SUM2RL=SUM2RL-COEFRL*CST2                                     **B 049
      SUM2IM=SUM2IM-COEFIM*CST2                                     **B 050
  100 CONTINUE                                                        B 051
      ARG=XKAYO*U1                                                    B 052
      CS1=COS(ARG)                                                    B 053
      SN1=SIN(ARG)                                                    B 054
      ARG=XKAYO*U2                                                    B 055
      CS2=COS(ARG)                                                    B 056
      SN2=SIN(ARG)                                                    B 057
      XI12RL=XKAYO*(SN2*SUM2RL-CS2*SUM2IM)                            B 058
```

```
      X1121M=XKAYO*(CS2*SUM2RL+SN2*SUM2IM)
      CST1=SN1*SUM1RL
      CST2=CS1*SUM1IM
      CST3=CS1*SUM1RL
      CST4=SN1*SUM1IM
      IF (U1.LT.0.) GO TO 200
      X111RL=XKAYO*(CST1-CST2)
      X111IM=XKAYO*(CST3+CST4)
      GO TO 300
  200 X111RL=XKAYO*(CST1+CST2)+2.*(CS1-1.+XKYOSQ*SUM)
      X111IM=XKAYO*(CST3-CST4)-SN1-SN1
  300 CONTINUE
      CST1=(XO/R)+1.
      CST2=CST1-2.
      XK11RL=CST1*CS1-X111RL
      XK111M=-CST1*SN1-X111IM
      XK12RL=CST2*CS2+X112RL
      XK121M=-CST2*SN2+X112IM
      SUM1RL=XK11RL+XK12RL
      SUM1IM=XK111M+XK121M
      XKREAL=.5*(CS*SUM1RL+SN*SUM1IM)
      XKIMAG=.5*(CS*SUM1IM-SN*SUM1RL)
      RETURN
C
C STEADY FORM OF KERNEL FUNCTION
C
  400 XKREAL=XO/R
      XKIMAG=0.
      RETURN
C
C SPECIAL FORM OF KERNEL FUNCTION AT YO=0
C
  500 XKREAL=CS
      XKIMAG=-SN
      RETURN
      END
```